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APPARENT HEADING ERROR INDUCED BY THE ROLL AND PITCH OF THE TOW--ETC(U)

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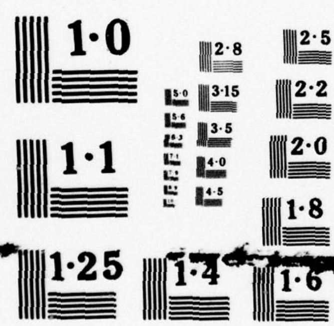
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U. S. NAVY UNDERWATER SOUND LABORATORY
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APPARENT HEADING ERROR INDUCED BY THE ROLL AND PITCH OF THE TOWED BODY IN A VDS SYSTEM.

by

Matthew F. Borg

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12 44p.

BACKGROUND

The difficulty and desirability of locating the towed body of a Variable Depth Sonar (VDS) system is discussed in Reference (a). Previously, there had been described possible methods for use in locating the towed body, with particular emphasis being given to an acoustic-type locator, Reference (b).

As a result, various proprietary proposals for the study and fabrication of a unit to be used for locating the towed body with respect to the towing ship, have been submitted, (References (c), (d), and (e)). These proposals are definitive in outlining (1) the characteristics of their respective systems and (2) the quantities to measure in order to obtain the heading of the towed body relative to the towing ship.

INTRODUCTION

The effect of the roll and pitch of the towing vessel on the heading information obtained with these systems is recognized as a possibility for error. However additional, or in many cases, existing instrumentation may be utilized to compensate for the towing ship motion and to feed the corrected information into a fire control computer.

However, the effect of the roll and pitch of the towed body on the heading information obtained with these systems is generally hypothesized not to be significant enough to cause appreciable errors. But in order to obtain such information as to the significance of the roll and pitch, much more elaborate electronic instrumentation must be produced,

since the presently installed pendulous type of indicators are not too reliable readout devices because of rapid inertia movement.

Thus, the proposals in References (c), (d), and (e) assume that the pitch and roll of the towed body will not cause an appreciable error in the heading information. If such errors are to occur, additional system components must be built and installed in order to compensate for the towed body's excursions through roll and pitch. Analyses, determining the effect of the towed body roll and pitch on the measurable quantities in each of the above systems, have not been performed.

PURPOSE OF STUDY

The present This memorandum evaluates the effect on a heading indicator of the towed body's motion through roll and pitch movements. The emphasis is on a heading indicator, because the mathematical presentation in this memorandum applies to a specific geometric configuration, which may or may not be used for actual measurements. The presentation is not a duplication of the measurable qualtities discussed in Reference (c), (d), or (e). For the systems in these References, actual calculations (or measurements) will have to be made on their geometric spaces in order to determine the effect of pitch and roll.

SIGNIFICANCE OF THE STUDY

The significance of the present memorandum is twofold:

a. A simple geometric solution has been obtained in which the quantitative error in heading is evaluated through a pitch and roll movement of the towed body. This heading error, to repeat, may not have any application to a presently proposed heading measuring system. The heading error is called an Apparent Heading Error (AHE), because the measuring yardstick, a unit vector $\bar{n}$, indicates a yaw angle, yet the towed body directional fixed heading axis, X_f, remains unchanged in direction, (see Fig 1a).

b. The qualitative conclusions obtained through this analysis may be applicable for all heading systems, thereby permitting a more intelligent discussion of the pitch and roll effect on the heading indicators of the towed body for any particular system.

THEORETICAL ANALYSIS

Assumptions

The following assumptions are made in the analysis:

a. The towing vessel is dynamically stable, rigid, and is compensating for wind and current effects in not deviating from its heading.

b. The system is composed of an acoustic projector on the towing vessel (point O) and two receiving hydrophones, A and B on the towed body. The acoustic projector is mechanically or electronically uncoupled from any rolling or pitching motion of the towing vessel. The receiving hydrophones are mounted on rigid supports. The hydrophones are situated athwartship ($\pm$ C) of the vertical axis plane (X_f, Z_f plane); either above or below ($\pm$ b) the direction-of-motion plane (X_f, Y_f plane), and either forward or aft ($\pm$ a) of the center of gravity (CG), (see Fig. 1a)).

c. The towed body initially faces in the same direction of motion as the towing vessel, and continues to face in that direction throughout its roll and pitch movement, i.e., the true angle of yaw is zero;

d. The water medium is homogeneous, hence;

e. The geometric plane formed by the vectors $\overline{OA}$ and $\overline{OB}$ from the acoustic projector to each of the receiving hydrophones remains undistorted during the pitch and roll motion of the towed body;

f. Inertia and depth effects on the response of the hydrophones are ignored; and,

g. The towline towpoint is situated at the center of gravity of the towed body.

Geometry

The basic geometry of the analyses is shown in Fig. 1a. When applicable, the nomenclature used is that recommended by Reference (f). The acoustic projector is located at Point O, the intersection of the towing vessel's fixed orthogonal axes X, Y, and Z. A left-handed screw convention is used for all axes. The X axis is the direction of motion the true heading direction. The vertical gravity axis is Z, downward. The Y axis is parallel to the water surface, see Fig. 1a.

The towed body is at some arbritrary location behind the projector on the towing vessel. The distance aft along the X axis to the center of gravity of the towed body is designated Trail (T). The distance athwartship along the Y axis is designated Cross-Trail (C_T). The vertical distance down, Depth (d).

The axes X_f, Y_f, and Z_f are located at the center of gravity (CG) of the fish, and are fixed in space. The fish axes are parallel to the towing vessel axes. The fore or aft distance to the receiving hydrophones A and B is $\underline{+a}$. The depth to the hydrophones is $\underline{+b}$. The offset athwartship distances are $\underline{+C}$.

Line vectors $\overline{OA}$ and $\overline{OB}$ define a plane connecting the acoustic projector to the hydrophones. The unit vector $\bar{n}$ is normal to this plane and is defined as being proportional to the vector crossproduct of OA and $\overline{OB}$. The Apparent Heading Error is obtained from an analysis on the direction of the unit vector $\bar{n}$ projections on the X, Y plane. The unit vectors in the X, Y, Z directions are $\bar{i}$, $\bar{j}$, $\bar{k}$, respectively.

Analysis

A. Zero Roll, Zero Pitch Case

The coordinates of the hydrophones are(Fig. 1a):

$$A = A(X_A, Y_A, Z_A) = ((T\pm a), (C_T+C), (d\pm b))$$
$$B = B(X_B, Y_B, Z_B) = ((T\pm a), (C_T-C), (d\pm b)) \qquad (1)$$

The unit vector $\bar{n}$ for a zero roll, zero pitch situation of the plane OAB is defined as the crossproduct of vectors $\overline{OA}$ and $\overline{OB}$, or:

$$\bar{n} = \frac{\overline{OA} \times \overline{OB}}{\text{mag.}} = \frac{[(C_T+C)(d\pm b)-(d\pm b)(C_T-C)]\bar{i} + [(T\pm a)(C_T-C)-(T\pm a)(C_T+C)]\bar{k}}{\text{magnitude}} \qquad (2)$$

The projection of this vector in the X-Y plane shows a component parallel to the Y axis. The angle γ of this component with the heading direction (X axis) is 0^o, or

$$\text{Tangent } \gamma = \frac{y \text{ coordinate}}{x \text{ coordinate}} = 0 \quad , \therefore \gamma = 0^\circ \qquad (3)$$

The Apparent Heading Error (A.H.E.) is defined as

$$\text{A.H.E.} = \tan^{-1} \quad y/x = \gamma \tag{4}$$

For a zero roll, zero pitch case, the Apparent Heading Error is therefore seen to be zero.

B. Finite Roll, Zero Pitch Case

If the hydrophones are rotated an angle θ (positive direction when hydrophone B decreases depth) about the X_f axis through the center of gravity point, a finite roll is introduced, (see Fig. 1b). The new coordinates of points $A(X_A, Y_A, Z_A)$ and $B(X_B, Y_B, Z_B)$ designated $A_1(X_{A_1}, Y_{A_1}, Z_{A_1})$, and $B_1(X_{B_1}, Y_{B_1}, Z_{B_1})$ after rotation are obtained from application of a matrix transformation (Reference (h)), as follows:

$$A_1 = \begin{bmatrix} X_{A_1} \\ Y_{A_1} \\ Z_{A_1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \tag{5}$$

$$B_1 = \begin{bmatrix} X_{B_1} \\ Y_{B_1} \\ Z_{B_1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} \tag{6}$$

Expanding equations (5) and (6) into Cartesian form, there is obtained:

$$\begin{aligned} X_{A_1} &= X_A \\ Y_{A_1} &= Y_A \cos\theta - Z_A \sin\theta \\ Z_{A_1} &= Y_A \sin\theta + Z_A \cos\theta \end{aligned} \tag{7}$$

and

$$\begin{aligned} X_{B_1} &= X_B \\ Y_{B_1} &= Y_B \cos\theta - Z_B \sin\theta \\ Z_{B_1} &= Y_B \sin\theta + Z_B \cos\theta \end{aligned} \tag{8}$$

Inserting the values of coordinates A and B from equation (1) into equations (7) and (8), the new vectors $\overline{OA}_1$ and $\overline{OB}_1$ can be written as:

$$\overline{OA}_1 = (T \pm a)\bar{i} + [(G_T + c)\cos\theta - (d \pm b)\sin\theta]\bar{j} + [(G_T + c)\sin\theta + (d \pm b)\cos\theta]\bar{k} \tag{9}$$

$$\overline{OB}_1 = (T \pm a)\bar{i} + [(G_T - c)\cos\theta - (d \pm b)\sin\theta]\bar{j} + [(G_T - c)\sin\theta + (d \pm b)\cos\theta]\bar{k} \tag{10}$$

The unit vector $\bar{n}_1$ defining the plane $OA_1 B_1$ is:

$$\bar{n}_1 = \frac{\overline{OA}_1 \times \overline{OB}_1}{mag} = \frac{1}{mag} \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ T \pm a & (G_T + c)\cos\theta - (d \pm b)\sin\theta & (G_T + c)\sin\theta + (d \pm b)\cos\theta \\ T \pm a & (G_T - c)\cos\theta - (d \pm b)\sin\theta & (G_T - c)\sin\theta + (d \pm b)\cos\theta \end{vmatrix} \tag{11}$$

Expansion of the determinant in equation (11) yields:

$$\begin{aligned} \bar{n}_1(mag) = \bar{i} \Big\{ &[(G_T + c)\cos\theta - (d \pm b)\sin\theta][(G_T - c)\sin\theta + (d \pm b)\cos\theta] \\ &- [(G_T - c)\cos\theta - (d \pm b)\sin\theta][(G_T + c)\sin\theta + (d \pm b)\cos\theta] \Big\} \\ - \bar{j} \Big\{ &(T \pm a)[(G_T - c)\sin\theta + (d \pm b)\cos\theta] - (T \pm a)[(G_T + c)\sin\theta + (d \pm b)\cos\theta] \Big\} \\ + \bar{k} \Big\{ &\cdots \Big\} \end{aligned} \tag{12}$$

Reduction of equation (12) gives:

$$\bar{n}_1(mag) = \bar{i}\,[2c(d \pm b)] + \bar{j}\,[2c(T \pm a)\sin\theta] + \bar{k}\,[\cdots] \tag{13}$$

From equations (4) and (13), the Apparent Heading Error for a Finite Roll, Zero Pitch case is:

$$AHE = \gamma = \tan^{-1} y/x = \tan^{-1}\left\{\frac{2c(T \pm a)\sin\theta}{2c(d \pm b)}\right\}$$

or:

$$AHE = \tan^{-1}\left[\frac{(1 \pm a/T)\sin\theta}{d/T \pm b/T}\right] \tag{14}$$

Equation (14), which represents the Apparent Heading Error, i.e., the angle between the projection of a unit vector of the acoustic plane OA_1B_1 on the X-Y plane, and to the X direction of heading, is presumably the heading error, which would be read in any measuring system if this unit vector were the measurable quantity. Equation (14) has been calculated on an IBM 704 computer for the following variation of parameters:

(a) At roll angles of 1°, 2°, 3°, 4°, 5°, and 6°; and
(b) At each roll angle:

$d/T = 1, 2, 3, 4, 5$, and 10; and

(c) At each d/t:

a/T and $\pm\, b/T$ simultaneously = 0.001, 0.01, 0.05, and 0.1.

The information from the computer was plotted automatically by The Electronics Associates 3440 Data Plotter. Figures (2) and (3), the results, show the angle of roll versus Apparent Heading Error (A.H.E.) at constant a/t and b/t with varying d/t, as obtained from equation (14).

C. Zero Roll, Finite Pitch Case

If the hydrophones are rotated at angle ϕ (positive direction when hydrophones A and B decrease in depth) about axis Y_f at the center of gravity point (CG), a finite pitch is introduced (see Fig. 1C). The new coordinates of points A and B, designated $A_2(X_{A_2}, Y_{A_2}, Z_{A_2})$ and $B_2(X_{B_2}, Y_{B_2}, Z_{B_2})$ after rotation, are obtained from application of a matrix transformation (Reference (n)) as follows:

$$A_2 = \begin{bmatrix} X_{A_2} \\ Y_{A_2} \\ Z_{A_2} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \qquad (15)$$

$$B_2 = \begin{bmatrix} X_{B_2} \\ Y_{B_2} \\ Z_{B_2} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} \qquad (16)$$

Expanding equations (5) and (16) into Cartesian form, there is obtained:

$$\begin{aligned} X_{A2} &= X_A \cos\phi + Z_A \sin\phi \\ Y_{A2} &= Y_A \\ Z_{A2} &= -X_A \sin\phi + Z_A \cos\phi \end{aligned} \tag{17}$$

and

$$\begin{aligned} X_{B2} &= X_B \cos\phi + Z_B \sin\phi \\ Y_{B2} &= Y_B \\ Z_{B2} &= -X_B \sin\phi + Z_B \cos\phi \end{aligned} \tag{18}$$

Inserting the values of coordinates A and B from equation (1) into equations (15) and (16), the new vectors $\overline{OA}_2$ and $\overline{OB}_2$ can be written as:

$$\overline{OA_2} = [(T \pm a)\cos\phi + (d \pm b)\sin\phi]\bar{i} + (C_T + c)\bar{j} + [\cdots]\bar{k} \tag{19}$$

$$\overline{OB_2} = [(T \pm a)\cos\phi + (d \pm b)\sin\phi]\bar{i} + (C_T - c)\bar{j} + [\cdots]\bar{k} \tag{20}$$

The unit vector $\bar{n}_2$ defining the plane OA_2B_2 is:

$$\bar{n}_2 = \frac{\overline{OA_2} \times \overline{OB_2}}{mag} = \frac{1}{mag}\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ (T \pm a)\cos\phi + (d \pm b)\sin\phi & C_T + c & -(T \pm a)\sin\phi + (d \pm b)\cos\phi \\ (T \pm a)\cos\phi + (d \pm b)\sin\phi & C_T - c & -(T \pm a)\sin\phi + (d \pm b)\cos\phi \end{vmatrix} \tag{21}$$

Expansion of the determinant in equation (21) yields:

$$\bar{n}_2(mag) = \bar{i}\left\{(C_T+c)\left[-(T\pm a)\sin\phi + (d\pm b)\cos\phi\right] - (C_T - c)\left[-(T\pm a)\sin\phi + (d\pm b)\cos\phi\right]\right\}$$

$$-\bar{j}\left\{\left[(T\pm a)\cos\phi + (d\pm b)\sin\phi\right]\left[-(T\pm a)\sin\phi + (d\pm b)\cos\phi\right] - \left[(T\pm a)\cos\phi + (d\pm b)\sin\phi\right]\left[-(T\pm a)\sin\phi + (d\pm b)\cos\phi\right]\right\} + \bar{k}\left\{\cdots\right\} \tag{22}$$

Reduction of equation (22) gives:

$$\bar{n}_2(mag) = \bar{i}\left\{2c\left[-(T\pm a)\sin\phi + (d\pm b)\cos\phi\right]\right\} + \bar{j}(0) + \bar{k}\left\{\cdots\right\} \tag{23}$$

From equations (4) and (23), the Apparent Heading Error for a zero roll, finite pitch case is:

$$AHE = \gamma = \tan^{-1} y/x = \tan^{-1}\frac{0}{x} = 0 \tag{24}$$

D. <u>Finite Roll, Finite Pitch Case</u>

If the hydrophones are simultaneously rotated an angle θ (positive direction defined in case B) and an angle ϕ (positive direction defined in case C), then both a finite roll and a finite pitch is introduced. Mathematically, the new coordinates of points A and B are obtained from superposition. However, in obtaining the new position of points A and B, a basic physical property of movement must be considered. This basic property is that the roll and pitch rotations are not commutative, i.e., by first rolling and then pitching, one does not obtain the same position by reversing the motion, i.e., first letting pitching occur and then rolling. This difficulty is overcome by considering <u>small</u> rotations to occur, i.e., replacing the cosine of the angle by one and the sine of the angle by the angle itself. In matrix formulation, the coordinates of the new positions A_3 (X_{A_3}, Y_{A_3}, Z_{A_3}) and B_3(X_{B_3}, Y_{B_3}, Z_{B_3}) of points A and B are:

$$A_3 = \begin{bmatrix} X_{A_3} \\ Y_{A_3} \\ Z_{A_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\theta \\ 0 & \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \phi \\ 0 & 1 & 0 \\ -\phi & 0 & 1 \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \tag{25}$$

and:

$$B_3 = \begin{bmatrix} X_{B3} \\ Y_{B3} \\ Z_{B3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\theta \\ 0 & \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \phi \\ 0 & 1 & 0 \\ -\phi & 0 & 1 \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} \tag{26}$$

Expanding equations (25) and (26) into Cartesian form, there is obtained:

$$\begin{aligned} X_{A3} &= X_A + Y_A[\theta\phi] + Z_A[\phi] \\ Y_{A3} &= X_A[0] + Y_A + Z_A[-\theta] \\ Z_{A3} &= X_A[-\phi] + Y_A[\theta] + Z_A \end{aligned} \tag{27}$$

and

$$\begin{aligned} X_{B3} &= X_B + Y_B[\theta\phi] + Z_B[\phi] \\ Y_{B3} &= X_B[0] + Y_B + Z_B[-\theta] \\ Z_{B3} &= X_B[-\phi] + Y_B[\theta] + Z_B \end{aligned} \tag{28}$$

In assuming small rotations, the product $\theta\,\phi$, which appears in the X coordinate, is assumed to be of a lower order of magnitude to either θ or ϕ, and hence can be neglected.

Inserting the values of coordinates A and B from equation (1) into equations (27) and (28), the new vectors $\overline{OA}_3$ and $\overline{OB}_3$ can be written as:

$$\begin{aligned} \overline{OA}_3 = \bar{i}\left[(T\pm a) + (d\pm b)\phi\right] &+ \bar{j}\left[(C_T+c) - (d\pm b)\theta\right] \\ &+ \bar{k}\left[-(T\pm a)\phi + (C_T+c)\theta + (d\pm b)\right] \end{aligned} \tag{29}$$

and

$$\begin{aligned} \overline{OB}_3 = \bar{i}\left[(T\pm a) + (d\pm b)\phi\right] &+ \bar{j}\left[(C_T-c) - (d\pm b)\theta\right] \\ &+ \bar{k}\left[-(T\pm a)\phi + (C_T-c)\theta + (d\pm b)\right] \end{aligned} \tag{30}$$

The unit vector $\bar{N}_3$ defining the plane OA_3B_3 is:

$$\bar{N}_3 = \frac{\overline{OA}_3 \times \overline{OB}_3}{mag} = \frac{1}{mag}\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ (T\pm a)+(d\pm b)\phi & (C_T+c)-(d\pm b)\theta & -(T\pm a)\phi+(C_T+c)\theta+(d\pm b) \\ (T\pm a)+(d\pm b)\phi & (C_T-c)-(d\pm b)\theta & -(T\pm a)\phi+(C_T-c)\theta+(d\pm b) \end{vmatrix} \tag{31}$$

Expansion of the determinant in equation (31) yields:

$$\begin{aligned} \bar{N}_3(mag) = \bar{i}\Big\{ & \big[(C_T+c)-(d\pm b)\theta\big]\big[-(T\pm a)\phi+(C_T-c)\theta+(d\pm b)\big] \\ & -\big[(C_T-c)-(d\pm b)\theta\big]\big[-(T\pm a)\phi+(C_T+c)\theta+(d\pm b)\big]\Big\} \\ -\bar{j}\Big\{ & \big[(T\pm a)+(d\pm b)\phi\big]\big[-(T\pm a)\phi+(C_T-c)\theta+(d\pm b)\big] \\ & -\big[(T\pm a)+(d\pm b)\phi\big]\big[-(T\pm a)\phi+(C_T+c)\theta+(d\pm b)\big]\Big\} \\ +\bar{k}\big\{ & \ldots\big\} \end{aligned} \tag{32}$$

Reduction of equation (32) gives:

$$\bar{N}_3(mag) = \bar{i}\big\{2c\left[(d\pm b)-(T\pm a)\phi\right]\big\} + \bar{j}\big\{2c(T\pm a)\theta\big\} + \bar{k}\{\ldots\} \tag{33}$$

From equations (4) and (33), the Apparent Heading Error for a finite roll, finite pitch case is:

$$AHE = \gamma = \tan^{-1} y/x = \tan^{-1}\left\{\frac{2c(T\pm a)\theta}{2c\left[(d\pm b)-(T\pm a)\phi\right]}\right\}$$

or (34)

$$AHE = \tan^{-1}\left[\frac{(1\pm a/T)\theta}{(d/T\pm b/T)-(1\pm a/T)\phi}\right]$$

Equation (34) represents the Apparent Heading Error for a small finite roll and small finite pitch case. For a pitch angle of 15°, the radian measureof the angle is approximately 1 per cent different from the sine of the angle, the cosine of 15 degrees differs approximately $3\frac{1}{2}$ per cent from one. A significant observation of equation (34) is the absence of both the distance C and cross-trail (C_T) distance in the linearized equation. A pitching situation by itself will introduce no Apparent Heading Error. But allow pitch to occur while roll is in process, and the pitch will effect the heading readings. When the pitch ϕ equals

zero, equation (34) reduces to equation (14) for small motions. When the roll Θ equals zero, equation (34) reduces to equation (24) for small motions.

Equation (34) has been calculated on an IBM 704 computer for the following variation of parameters:

(a) for $a/T = +0.03$, and $b/T = +0.03$
   (1) Θ ranges from 1°, 2°, 3°, 4°, 5°, and 10°.
   (2) at each Θ, $\emptyset$ ranges from 0°, 5°, 10°, and 15°.
   (3) at each $\emptyset$, $d/T = 1, 2, 3, 4, 5$, and 10.
(b) Repeating (a) for $a/T = +0.03$ and $b/T = -0.03$
(c) Repeating (a) for $a/T = -0.03$ and $b/T = +0.03$
(d) Repeating (a) for $a/T = -0.03$ and $b/T = -0.03$
(e) Repeating (a) for $a/T = +0.06$ and $b/T = +0.06$
(f) Repeating (a) for $a/T = -0.06$ and $b/T = -0.06$

The information from the computer was plotted automatically by the Electronic Associates 3440 Data Plotter. The results are shown in Figures (4) to (17), as the angle of roll versus Apparent Heading Error (A.H.E.) at constant d/T; a/T; b/T for varying angles of pitch.

Description and Use of Graphs

Figures (2) and (3) have as their abscissa the angle of roll in degrees, and for their ordinate, the Apparent Heading Error in radians, with an inserted table of degrees versus radians up to 15 degrees. Figure (4) to (17) have as their abscissa the angle of roll in radians, with an insert of degrees adjacent to the radian measure from 0 to 10 degrees and for their ordinate the Apparent Heading Error in radians, with an inserted table of degrees versus radians from 0 to 15 degrees.

Figures (2) and (3) cover the case B in which roll is the only action experienced by the towed body. Each of these figures is plotted under constant a/T (ratio of fore or aft distance to the receiving hydrophones, a, to the trail distance T) and constant b/T (ratio of distance to the hydrophones, b, to the trail distance, T), where the distances a and b are with respect to the towed body axes; (see Figure (1)). In Figures (2) and (3), the ratio of the depth of the towed body, d, to the trail distance, T, is varied from 1 through 10. As an illustration of their use:

Assume that the a/T and b/T ratios are both ± 0.001 (Figure (2)), and the angle of roll is 5 degrees; hence, the Apparent Heading Error for various values of d/T are as follows:

A.H.E. = 0.08694* radians = 4.98 degrees, at d/T = 1;
A.H.E. = 0.043562 radians = 2.50 degrees, at d/T = 2;
A.H.E. = 0.02906 radians = 1.67 degrees, at d/T = 3;
A.H.E. = 0.021802 radians = 1.25 degrees, at d/T = 4;
A.H.E. = 0.01744 radians = 1.00 degrees, at d/T = 5; and
A.H.E. = 0.00872 radians = 0.05 degrees, at d/T = 10

Nearly identical values are obtained from Figure (3), for a/T = b/T = ±0.1. An interpretation of the preceding tabluation would be: As the towed body experiences a constant roll as it falls deeper into the ocean, then the Apparent Heading Error decreases.

Figures (4) to (17) show the effects of simultaneous pitch and roll. Figures (4) to (7) maintain constant hydrophone location at a/T = b/T = +0.03, and illustrates the effect of changing depth with constant trail distance, on four different pitch angles, Ø, equal to 0, 5, 10, and 15 degrees. The location of the hydrophones in Figures (4) to (7) are arbitrarily assumed to be +0.03 trail units aft and below the center of gravity of the towed body, see Figure (1a). As an illustration of the use of Figures (4) to (7):

Assume that the simultaneous angle of roll and pitch are both 5 degrees (0.08727 radians); hence from the Figures (4) to (7), the Apparent Heading Errors are as follows:

A.H.E. = 0.09532 radians = 5.46 degrees, at d/T = 1;
A.H.E. = 0.04630 radians = 2.65 degrees, at d/T = 2;
A.H.E. = 0.03056 radians = 1.75 degrees, at d/T = 3; and
A.H.E. = 0.00904 radians = 0.52 degrees, at d/T = 10.

An interpretation of the preceding tabulation would be: As the towed body experiences a constant roll and pitch of 5 degrees as it falls deeper into the ocean, then the Apparent Heading Error decreases. In comparison with the previous tabulation for finite roll, zero degrees angle of pitch case, it is seen that at a constant d/T, the effect of pitch is to increase the Apparent Heading Error.

A complete analysis of Figures (2) to (17) is made in the Results and Discussion section.

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* This and subsequent values were taken from the computed results.

RESULTS AND DISCUSSION

Both the zero roll, zero pitch case (Case A) and the zero roll, finite pitch case (Case C) contribute zero Apparent Heading Error into the directionality of the vector used in the geometric analyses. The results from Case A are, however, trivial.

The results from Case C are interesting if identified with a measuring system of an actual towed body. If the medium is assumed homogeneous and the towed body has a plane of symmetry in its longitudinal vertical centerline plane, any pitching motion will not excite additional movements around or laterally to the other axes. This action is in contradistinction to rolling, since the action of rolling will tend to excite a pitching motion through the change in the center of buoyance, (see Reference (g)). The roll phenomenon, both in a pure roll and a combined roll-and-pitch motion, contributes an Apparent Heading Error to the measuring system, (see further discussion below). The pure pitching motion is a desirable characteristic of the towed body insofar as heading errors are concerned; therefore, if a feasible anti-rolling device could be incorporated into the towed body , theoretically no Apparent Heading Error would occur.

The zero pitch, finite roll case (Case B) has been calculated from equation (14) for a range of depth-to-trail ratios, for two different hydrophone locations: $a/T = b/T = \pm 0.001$ and ± 0.1. These results are shown plotted in Figures (2) and (3). The intermediate hydrophone locations at ± 0.01 and ± 0.05 were also computed, but these results differed very minutely from Figures (2) and (3), and consequently were not plotted.

The following qualitative observations are induced from the information presented in Figures (2) and (3):

(a) For a range of d/T from 1 to 10, and an angle of roll range from 0 to 6 degrees, (providing symmetry is preserved) there is no significance in the location of the hydrophones either forward, aft, below, or above the center of gravity of the body;

(b) For a constant d/T, the A.H.E. <u>increases</u> as the angle of roll <u>increases</u>;

(c) For a constant angle of roll, the A.H.E. <u>increases</u> as the d/T <u>decreases</u>, consequently;

(d) For a constant angle of roll and trail, as the depth of the towed body <u>increases</u>, the A.H.E. <u>decreases</u>; or

(e) For a constant angle of roll and depth, as the trail distance <u>decreases</u>, the A.H.E. <u>decreases</u>.

The asymptotic position of the towed body leads to the following A.H.E.:

From equation (14):

$$\lim_{T \to 0} AHE = \lim_{T \to 0} \tan^{-1}\left[\frac{(T \pm a)\sin\theta}{d \pm b}\right] = \tan^{-1}\left[\frac{\pm \sin\theta}{d/a \pm b/a}\right] \tag{35}$$

since $d/a \gg b/a$

$$\lim_{T \to 0} AHE = \tan^{-1}\left[\frac{\pm \sin\theta}{d/a}\right] \tag{35a}$$

Equation (35a) shows that for (a) a zero trail; (b) the towed body at a constant roll; and (c) the position of the hydrophone is fixed at some point relative to the center of gravity of the towed body, the A.H.E. <u>decreases</u> as the depth increases.

The foregoing observations are summarized in Table I and can be restated succinctly as follows:

For increasing roll with zero pitch:

The shallower the towed body and the farther directly aft with respect to the towing vessel, the larger the A.H.E.; or conversely, the deeper the towed body and the nearer directly aft with respect to the towing vessel, the lesser the A.H.E.

The following quantitative observations are found in Figures (2) and (3):

(a) For $a/T = b/T = \pm 0.001$, at a $d/T = 1$, the A.H.E. is directly related to the angle of roll, i.e., at a roll angle of 6 degrees, the A.H.E. is 6 degrees. As the d/T drops to 10, at a roll angle of 6 degrees, the A.H.E. drops to approximately one-half a degree.

(b) For $a/T = b/T = \pm 0.1$; at a $d/T = 1$, the A.H.E. is still directly related to the angle of roll. The influence of hydrophone location is very marginal. For the other d/T values, the difference in A.H.E. between those shown in Figures (2) and (3) are slight, differing at the angle of roll of 6 degrees and a $d/T = 10$, by only 0.0015 radians or 0.05 degrees.

Previously, the cross-coupling motion of roll and pitch had been mentioned. Consequently, the finite roll, finite pitch case (Case D), equation (34) is especially important in its description of the A.H.E. resulting during the tow.

Figures (4) through (7), show the bounded values of A.H.E. versus roll angle 0 to 10 degrees, through pitching angles of 0 to 15 degrees at a/T = b/T = 0.03,i.e., at receiving hydrophones placed 0.03 trail units aft and below the towed body's center of gravity, see Figure (1a). The following qualitative observations are obtained from Figures (4) through (7):

(a) For a constant d/T, the A.H.E.:

1. Increases as the angle of roll increases,
2. Increases as the angle of pitch increases, expect at large values of d/T, e.g., d/T = 10, where the effect of a change in pitch is negligible, (Figure (7));

(b) For a constant angle of roll, the A.H.E.:

1. Increases as the angle of pitch increases, again except for large values of d/T;
2. Decreases as the d/T ratio increases.

(c) For a constant angle of pitch, the A.H.E.:

1. Increases as the angle of roll increases;
2. Decreases as the d/T ratio increases.

Figures (8) and (9) depict the two end conditions for d/T, i.e., d/T = 1 and d/T = 10 for hydrophones placed 0.03 trail units aft and above the towed body's center of gravity. Figures (10) and (11) depict the end conditions for d/T, for hydrophones placed 0.03 trail units forward and below the towed body's center; Figures (12) and (13) for hydrophones 0.03 trail units forward and above; Figures (14) and (15) for hydrophones 0.06 trail units aft and below; and Figures (16) and (17) for hydrophones 0.06 trail units forward and above the towed body's center of gravity.

For each constant set of a/T and b/T curves between Figures (8) and (17), there are the same qualitative conclusions shown above for Figures (4) through (7). Additional qualitative conclusions relative to the fastening of the hydrophones on the towed body, (providing symmetry is preserved) are as follows; AT constant d/T, angle of roll and pitch:

(a) For hydrophones placed aft of the towed body's center of gravity:

1. Larger A.H.E. will occur for hydrophones positioned above compared to below (see Figures (4) and (8)); and

(b) For hydrophones placed forward of the towed body's center of gravity:

1. Larger A.H.E. errors will occur for hydrophones positioned above compared to below (See Figures (10) and (12)).

However, the magnitude of difference in A.H.E. between hydrophone locations tends to diminish that further forward or aft the hydrophones are placed from the towed body's center of gravity. (See Figures (14) and (16)). The foregoing observations are summarized in Table II and can be restated as follows:

For both increasing roll and pitch:

The shallower the towed body and the further directly aft with respect to the towing vessel and with the receiving hydrophones placed above the towed body's center of gravity, the larger the A.H.E.; or conversely, the deeper the towed body and the nearer directly aft with respect to the towing vessel and with the receiving hydrophones placed below the towed body's center of gravity, the lesser the A.H.E.

Quantitative observations from Figures (4) through (17) are:

(a) For a constant angle of roll at 10 degrees; a constant angle of pitch of 15 degrees and d/T = 1, the A.H.E. as a function of hydrophone location is as follows:

1. a/T = b/T = 0.03, A.H.E. = 0.23216 radians* = 13.3 degrees, Figure (4)
2. a/T = +0.03; b/T = -0.03, A.H.E. = 0.25126 radians = 14.4 degrees, Figure (8)
3. a/T = -0.03; b/T = +0.03, A.H.E. = 0.21478 radians = 12.3 degrees, Figure (10)
4. a/T = b/T = -0.03, A.H.E. = 0.23216 radians = 13.3 degrees Figure (12)

---

* This and subsequent values taken from computed values rather than from the graphs.

5. $a/T = b/T = +0.06$, A.H.E. = 0.23216 radisna = 13.3 degrees, Figure (14)
6. $a/T = b/T = -0.06$, A.H.E. = 0.23216 radians = 13.3 degrees, Figure (16)

The detrimental effect of locating the receiving hydrophones above the towed body's center of gravity (i.e. a minus b/T location) is evident, except for hydrophones placed further forward or aft. (i.e. > a/T).

(b) For a constant angle of roll at 10 degrees; a constant angle of pitch of 15 degrees and $d/T = 10$, the A.H.E. as a function of hydrophone location is as follows:

1. $A/T = b/T = +0.03$, A.H.E. = 0.01842 radians = 1.05 degrees, Figure (7)
2. $a/T = +0.03$; $b/T = -0.03$, A.H.E. = 0.01853 radians = 1.06 degrees, Figure (9)
3. $a/T = -0.03$, $b/T = +0.03$, A.H.E. = 0.01732 radians = 0.99 degrees, Figure (11)
4. $a/T = b/T = -0.03$, A.H.E. = 0.01742 radians = 1.00 degrees, Figure (13)
5. $a/T = b/T = +0.06$, A.H.E. = 0.01891 radians = 1.08 degrees, Figure (15)
6. $a/T = b/T = -0.06$, A.H.E. = 0.01692 radians = 0.97 degrees, Figure (17)

Again the effect of locating the hydrophones above the towed body's center of gravity is shown.

One asymptotic solution to equation (34) is quite interesting:

$$\lim_{T \to \infty} AHE = -\tan^{-1}\left(\theta/\phi\right) \qquad (36)$$

Equation (36) indicates that for very large trail distances, the A.H.E. varies directly as the angle of roll, and inversely as the angle of pitch. But, equal angles of pitch and roll will always give a 45 degree angle error in A.H.E. This value of A.H.E. lies beyond the range of linearization consistent with the transformation equation (25) and (26); however, in Equation (36) the trend in A.H.E. is indicated, namely: keeping the trail distance and roll angle as small as possible.

Equation (36) also shows that for very large trail distance, pitch tends to reduce the A.H.E. However, the relative magnitudes of pitch and

roll are important, viz: regardless how large or small in magnitude pitch is, when nearly equal or smaller in magnitude than roll, excessive A.H.E. will occur.

Quantitative values comparing the results from the Finite Roll, zero pitch case (Case B), and the Finite Roll, Finite Pitch Case (Case D), are shown in Table III. It is seen that for the lower value of d/T = 1, the effect of pitch is to increase the A.H.E., but as the trail becomes smaller or the depth becomes larger, the influence of pitch is to reduce the A.H.E. slightly from the unpitched case.

CONCLUSIONS

From the analysis of an acoustical type system designed for geometrically locating a towed body with respect to the towing vessel, for the magnitudes of possible towed body roll, pitch, and yaw angle deviations considered, i.e., ±10; ±15; and 0 degrees respectively, it is concluded that:

1. Quantitatively, the Apparent Heading Error (which is defined as a spurious signal in the towed body heading indicator measuring device) can, under a condition of tow inducing pure roll of the towed body, be on the same order of magnitude as the roll angle. If, additionally, pitching were to occur, the Apparent Heading Error would further increase;

2. For the Finite Roll, Zero Pitch Case, the shallower the towed body and the further directly aft with respect to the towing vessel, the larger the Apparent Heading Error, or conversely, the deeper the towed body and the closer directly aft with respect to the towing vessel, the lesser the Apparent Heading Error;

3. For the Finite Roll, Zero Pitch Case, the Apparent Heading Error decreases:

(a) as the angle of roll decreases and the depth to trail ratio remains constant; and

(b) as the depth to trail ratio increases and the angle of roll remains constant.

4. For the Finite Roll, Zero Pitch Case, the Apparent Heading Error is slightly influenced by the fore and aft or up and down location

of the receiving hydrophones with respect to the towed body's center of gravity, and is independent of the cross-trail location of the fish, and the athwartship location of the receiving hydrophones with respect to the towed body's center of gravity.

5. For the Finite Roll, Finite Pitch Case, the shallower the towed body, the further directly aft with respect to the towing vessel and with the receiving hydrophones placed above the towed body's center of gravity, the larger the Apparent Heading Error, or conversely, the deeper the towed body and the closer directly aft with respect to the towing vessel and with the receiving hydrophones placed below the towed body's center of gravity, the lesser the Apparent Heading Error.

6. For the Finite Roll, Finite Pitch Case, the Apparent Heading Error decreases:

(a) As the angle of roll decreases and the d/T, depth to trail ratio and angle of pitch are constant;
(b) As the angle of pitch decreases, and the d/T, depth to trail ratio and angle of roll are constant except at larger values of d/T (e.g., d/T = 10) where the effect of a change in pitch is negligible;
(c) As the receiving hydrophones are positioned below the towed body's center of gravity in preference to above. However, as the distance forward or aft (a/T) of the hydrophones is increased, the relative positioning either above or below tends towards no longer significantly effecting the Apparent Heading Error.

7. For the Finite Roll, Finite Pitch Case, as the receiving hydrophones are brought closer in the longitudinal axis direction to the towed body's center of gravity, the Apparent Heading Error is effected by the hydrophones being either above or below the gravity point. The Apparent Heading Error is independent of the Cross-trail location of the fish, and the athwartship location of the receiving hydrophones with respect to the gravity point.

MATTHEW F. BORG
Mechanical Engineer

TABLE I

CASE	d/T*	ANGLE OF ROLL θ	APPARENT HEADING ERROR
1	$\longrightarrow 0$	constant	increases
2	$\longrightarrow \infty$	constant	decreases
3	constant	increases	increases
4	constant	decreases	decreases

Table I - Finite Roll, Zero Pitch, Case B, Apparent Heading Error and its relationship to Fish depth (d), Trail (t) and Roll Angle (θ)

* The symbols $\longrightarrow 0$ and $\longrightarrow \infty$, imply "approaching zero and approaching infinity," respectively.

TABLE II

HYDROPHONES AT ANY LOCATION RELATIVE TO TOWED BODY'S CENTER OF GRAVITY				
Case	d/T	θ	ϕ	A.H.E.
1	<	>	>	>
2	C	>	C	>
3	C	C	>	>
4	C	>	<	C
5	>	C	C	<
6	>	>	C	<
7	>	C	>	<

NOTE: Magnitude wise, lower values of A.H.E. are experienced for hydrophones located below the axii containing the towed body's center of gravity.

Table II - Finite Roll, Finite Pitch Case (Case D), Apparent Heading Error and its Relationship to Fish Depth (d), Trail (t), Roll(θ), and Pitch (ϕ).

> = increasing
< = decreasing
C = constant

TABLE III

d/T	a/T	b/T	Roll, θ (degrees)	Pitch, Ø degrees	A.H.E. radians*	degrees	Fig. No.	Case
1	±0.001	±0.001	5	0	0.08694	4.98	2	B
1	±0.1	±0.1	5	0	0.08694	4.98	3	B
1	+0.03	+0.03	5	15	0.11767	6.74	4	D
1	+0.03	-0.03	5	15	0.12765	7.31	8	D
1	-0.03	-0.03	5	15	0.11767	6.74	12	D
1	±0.06	±0.06	5	15	0.11767	6.74	14,16	D
10	±0.001	±0.001	5	0	0.00872	0.50	2	B
10	+0.1	+0.1	5	0	0.00949	0.54	3	B
10	+0.03	+0.03	5	15	0.00920	0.53	7	D
10	+0.03	-0.03	5	15	0.00927	0.53	9	D
10	-0.03	-0.03	5	15	0.00871	0.50	13	D
10	±0.06	±0.06	5	15	0.00846	0.48	15,17	D

* Values taken from Computed results.

Table III - Quantitative Comparisons between the Finite Roll, Zero Pitch Case (Case B) and the Finite Roll, Finite Pitch Case (Case D). (Angle of Roll is 5 degrees, and angle of pitch is 15 degrees.)

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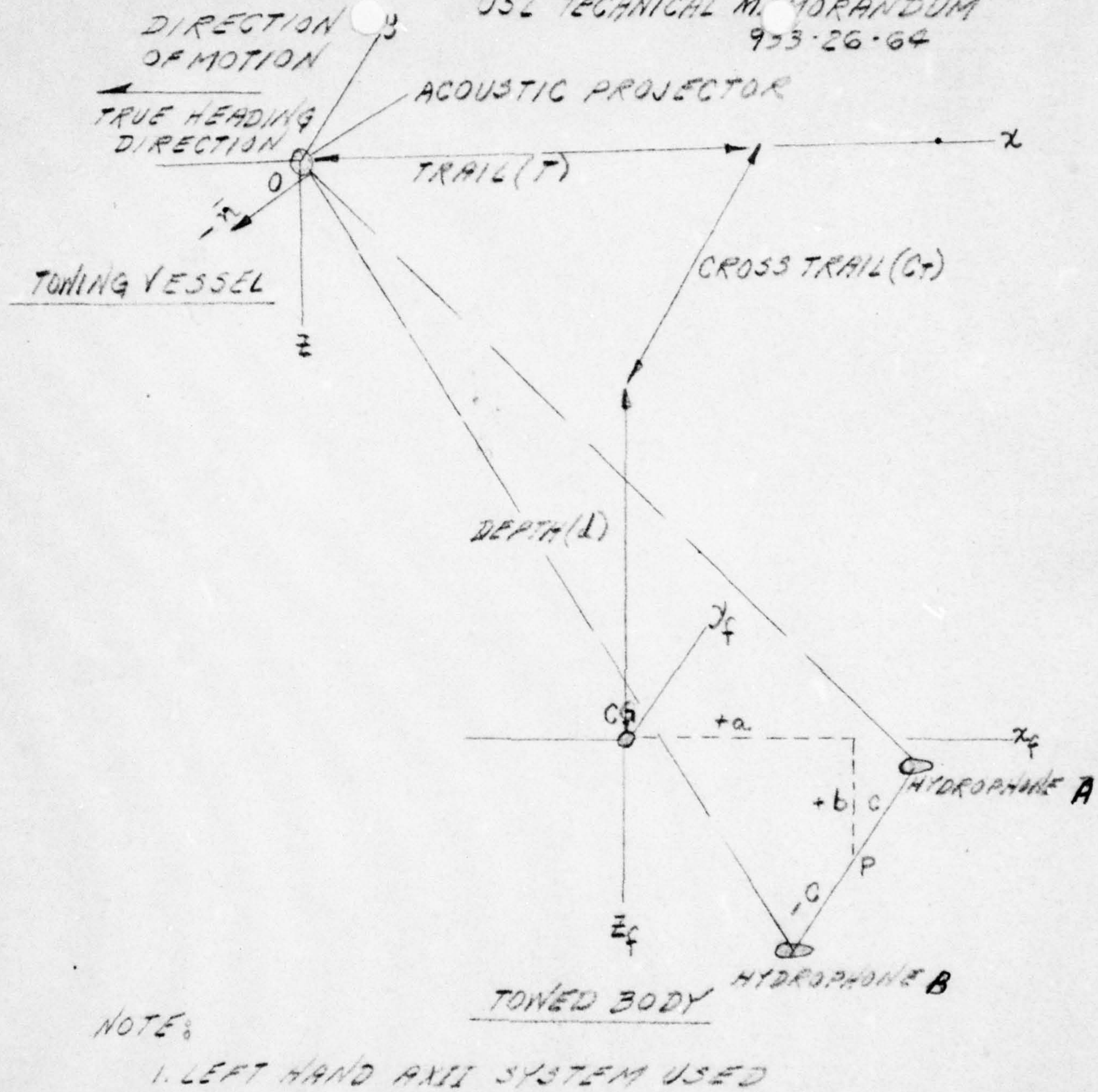
(F) A Nomenclature for Stability and Control, F. H. Imlay, Taylor Model Basin Report 1319, May 1959.

(G) H. E. Rossell and L. B. Chapman, Editors, Principals of Naval Architecture, Volume Two, Society of Naval Architects and Marine Engineers, New York, N. Y., 1939, Chapter 1, Section 8.

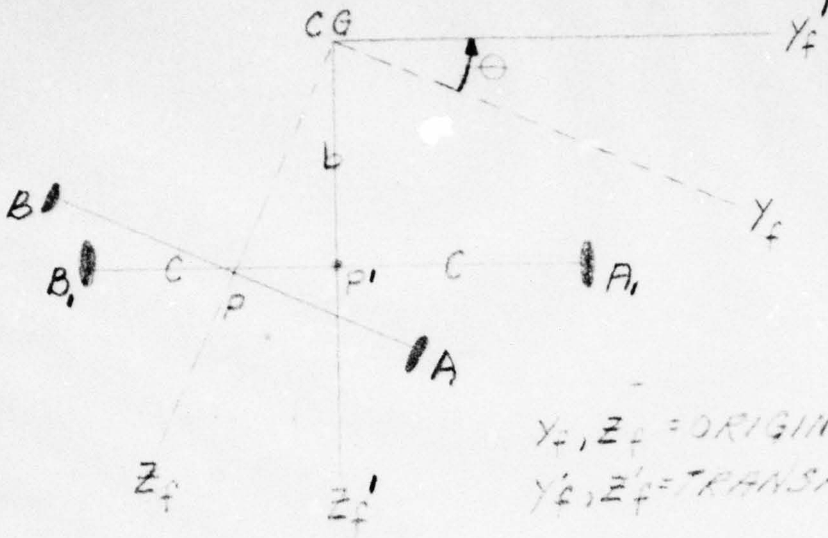
(H) D. E. Rutherford, Classical Mechanics, Interscience Publishers, New York, 1951, pp. 150-153.



NOTE:

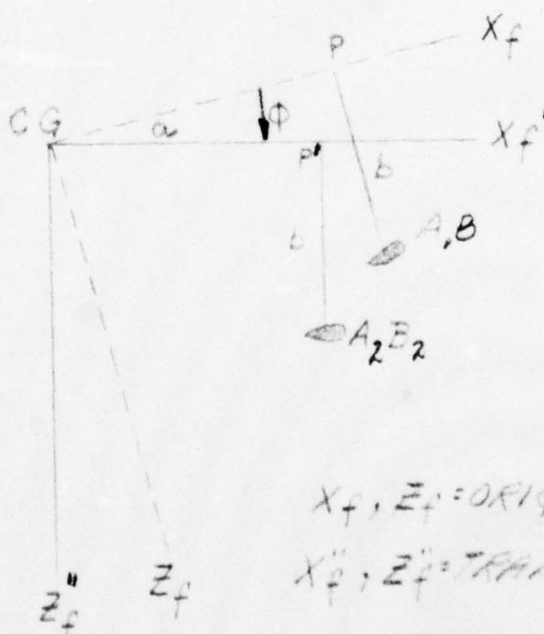
1. LEFT HAND AXII SYSTEM USED
2. AXII x, y, z ARE FIXED WITH RESPECT TO THE TOWING VESSEL AND THE WATER SURFACE
3. AXIS x_f, y_f, z_f ARE FIXED WITH RESPECT TO THE TOWED BODY CENTER OF GRAVITY

FIGURE-1a GEOMETRIC LOCATION OF TOWED BODY AND TOWING VESSEL



Y_f, Z_f = ORIGINAL AXII
Y_f', Z_f' = TRANSFORMED AXII

FIGURE-1b CHANGE IN HYDROPHONE COORDINATES THROUGH A ROLL ANGLE Θ



X_f, Z_f = ORIGINAL AXII
X_f'', Z_f'' = TRANSFORMED AXII

FIGURE-1c CHANGE IN HYDROPHONE COORDINATES THROUGH A PITCH ANGLE Φ

Figure 2 - ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT a/T = b/T for VARYING d/T

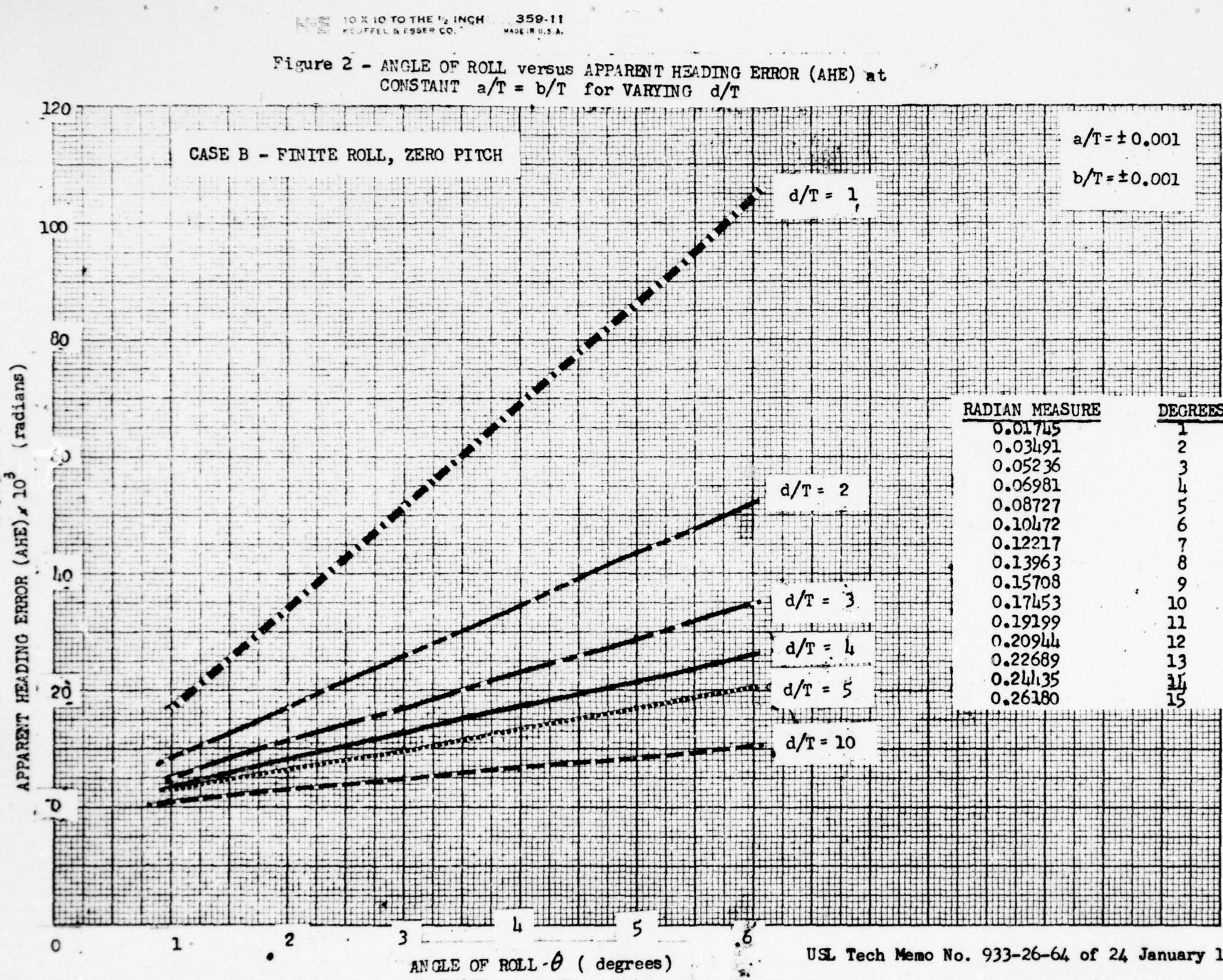
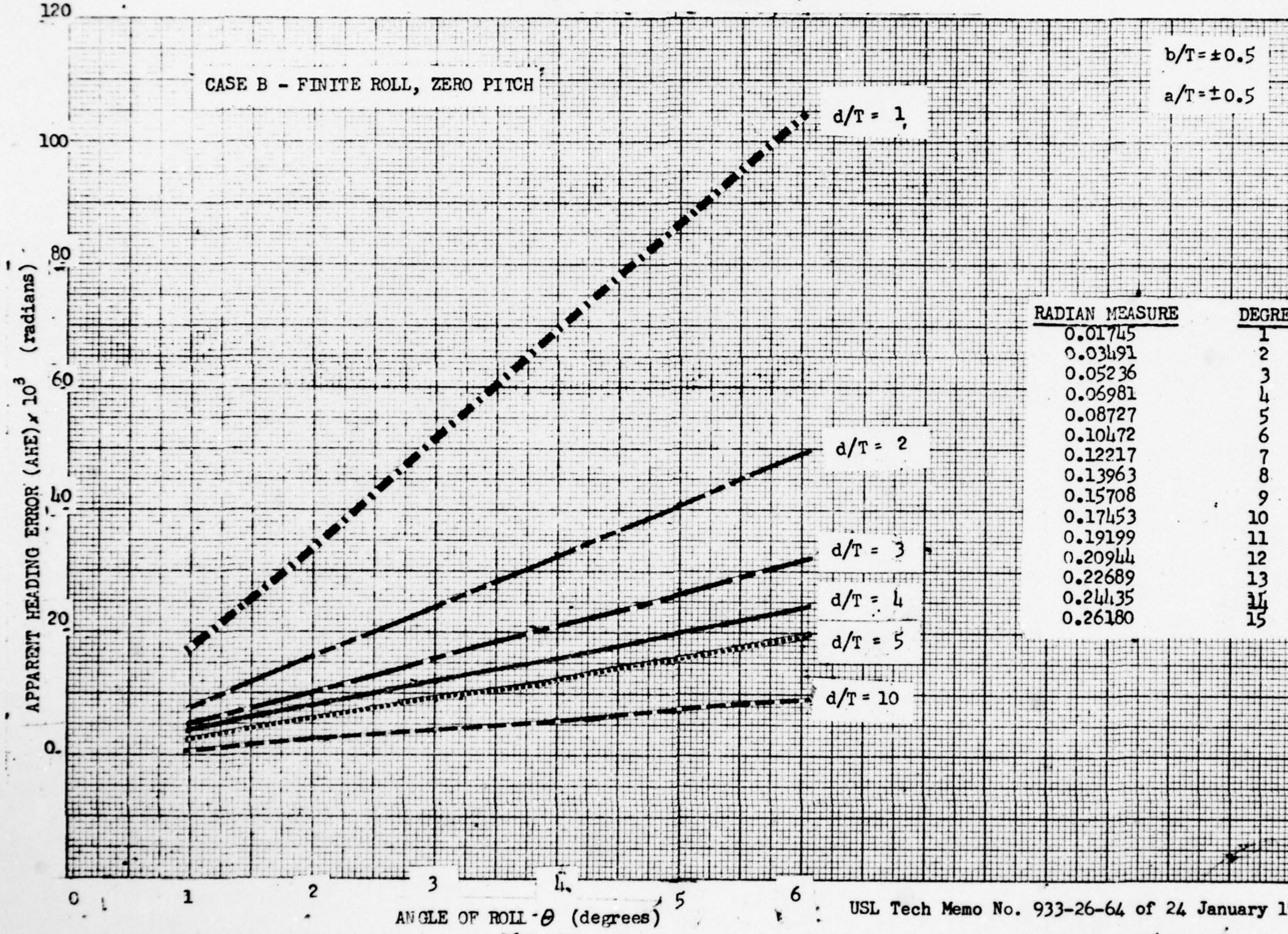
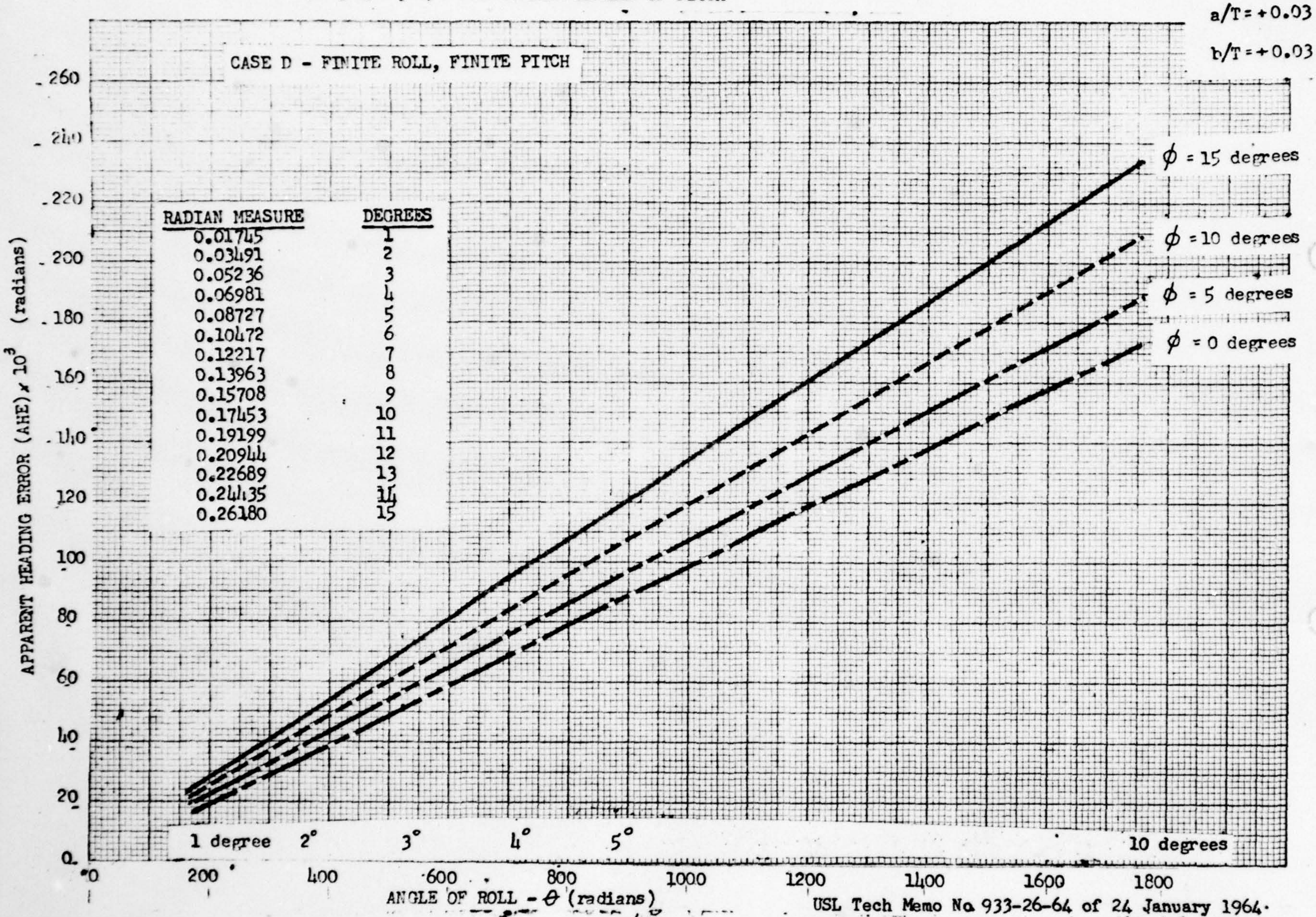


Figure 3 - ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT a/T = b/T for VARYING d/T



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

Figure 4 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH



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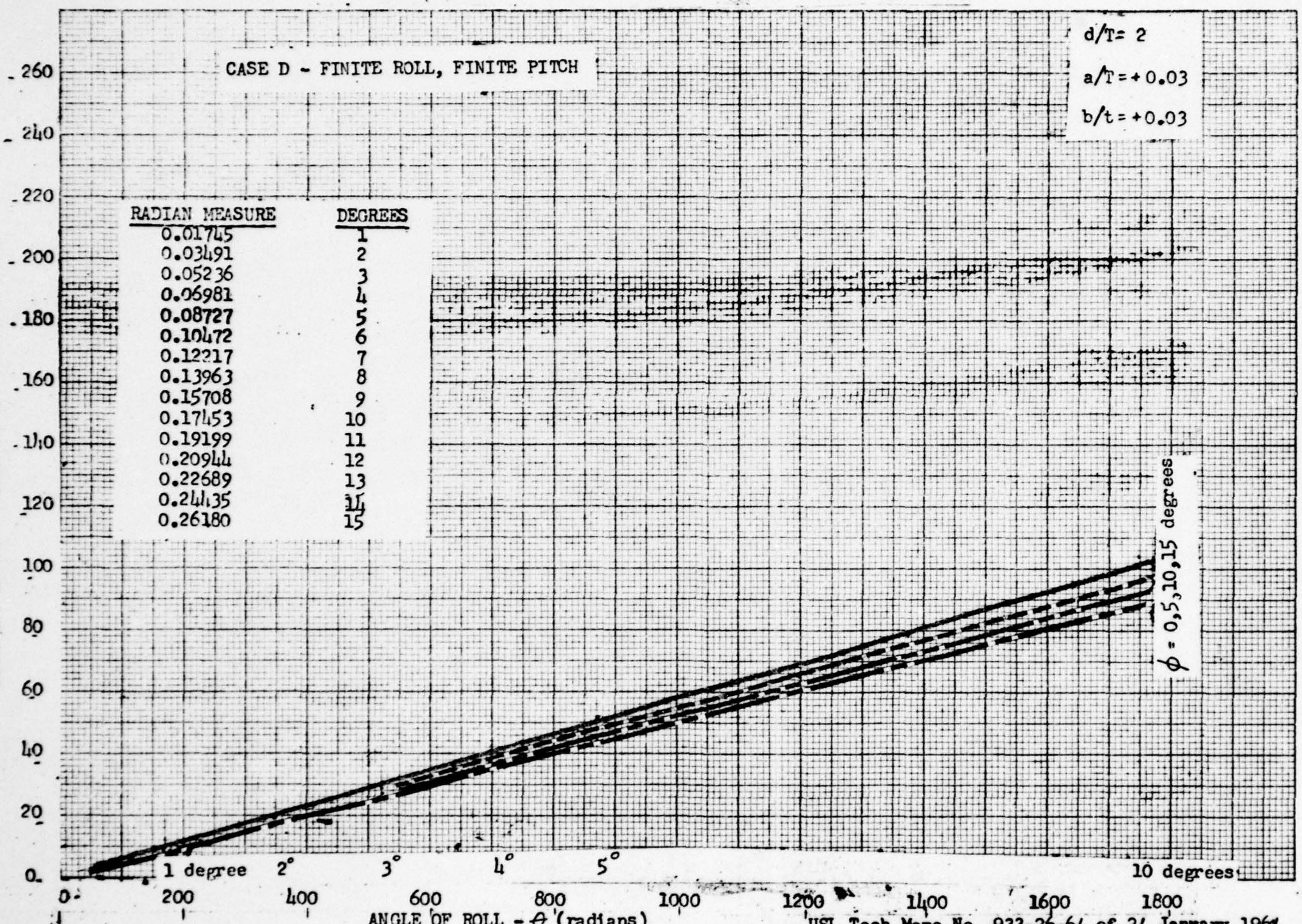
10 X 10 TO THE ½ INCH 359-11
KEUFFEL & ESSER CO. MADE IN U.S.A.

Figure 5 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

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Figure 6 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

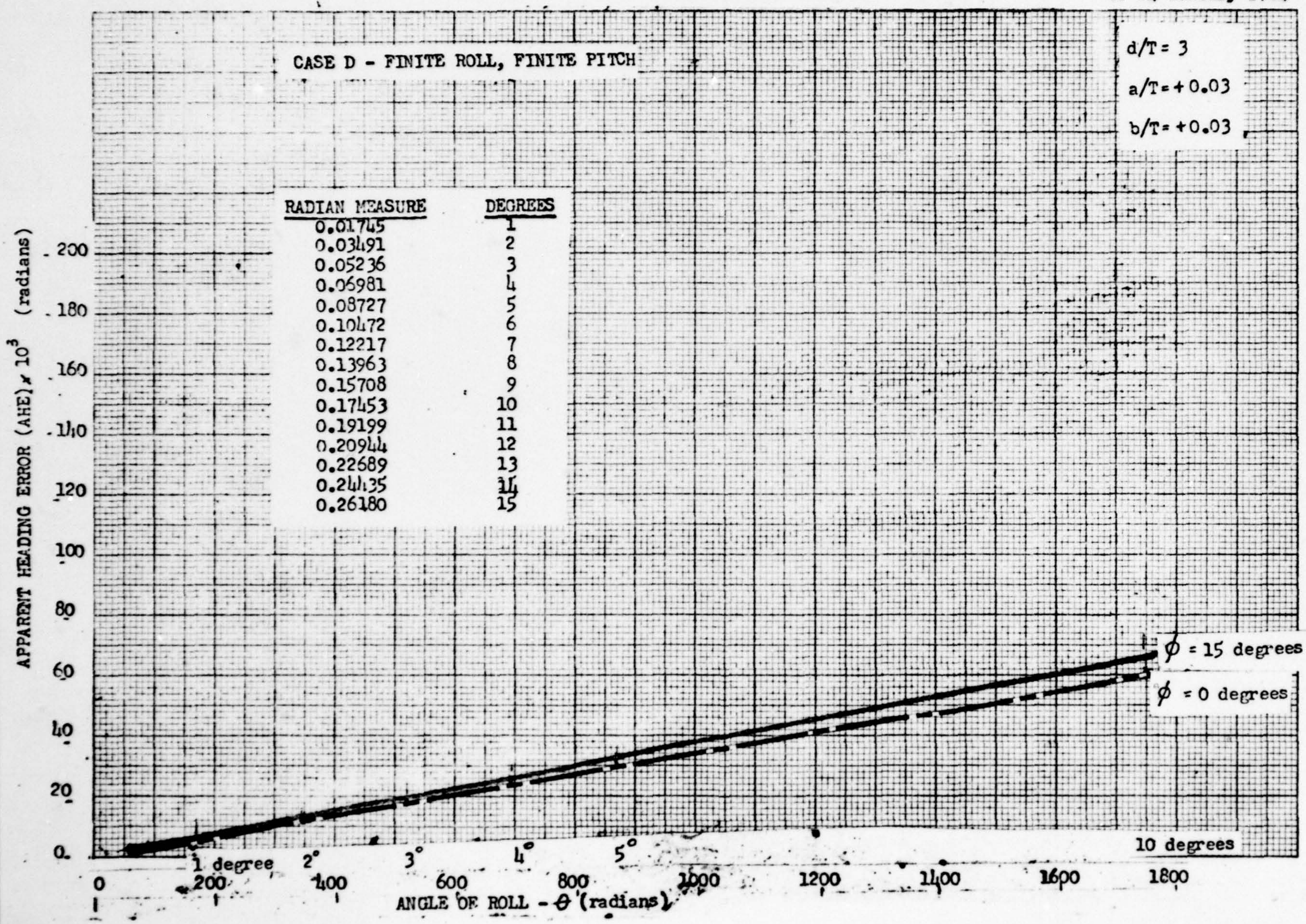

Figure 7 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

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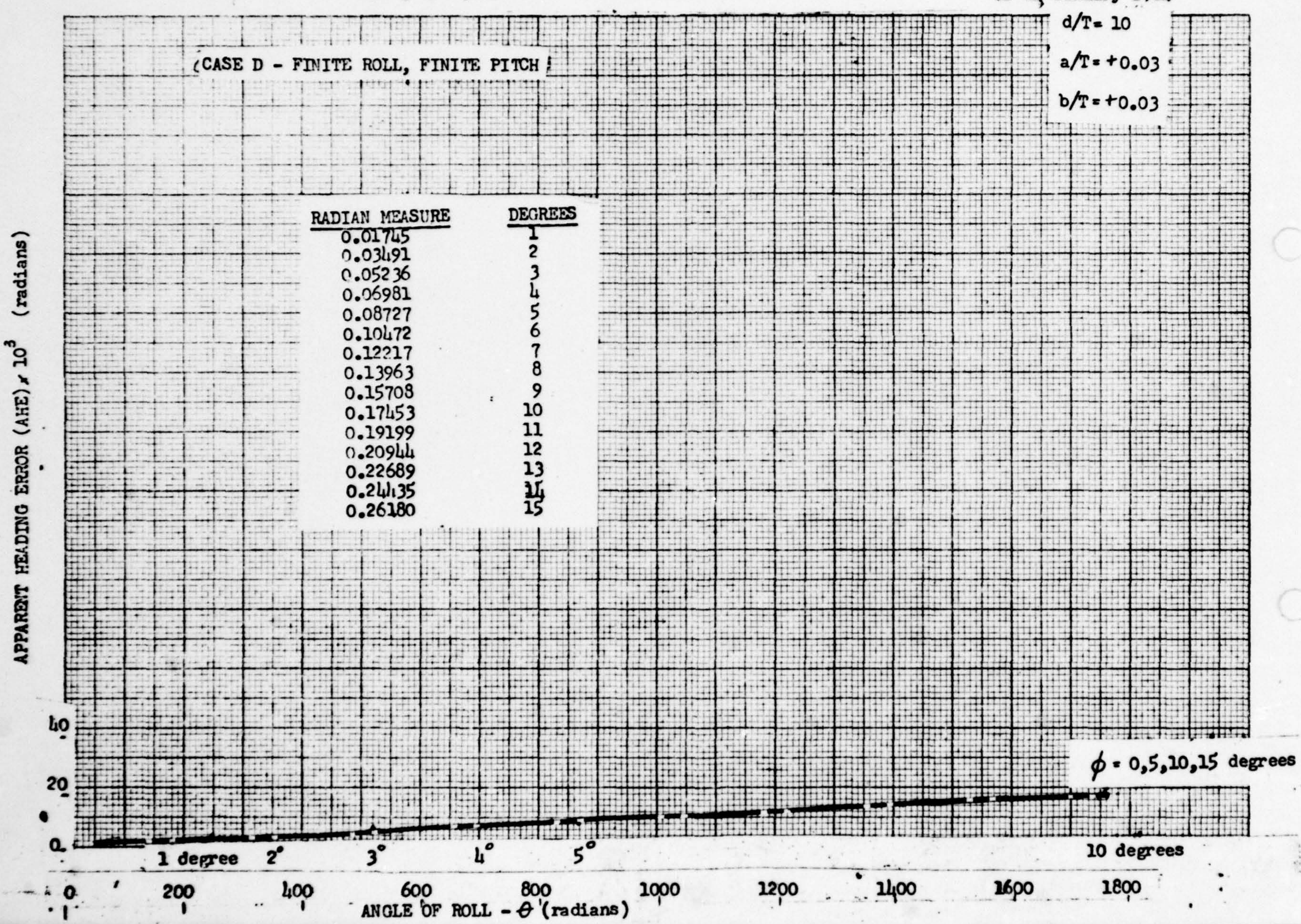

RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

Figure 8 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

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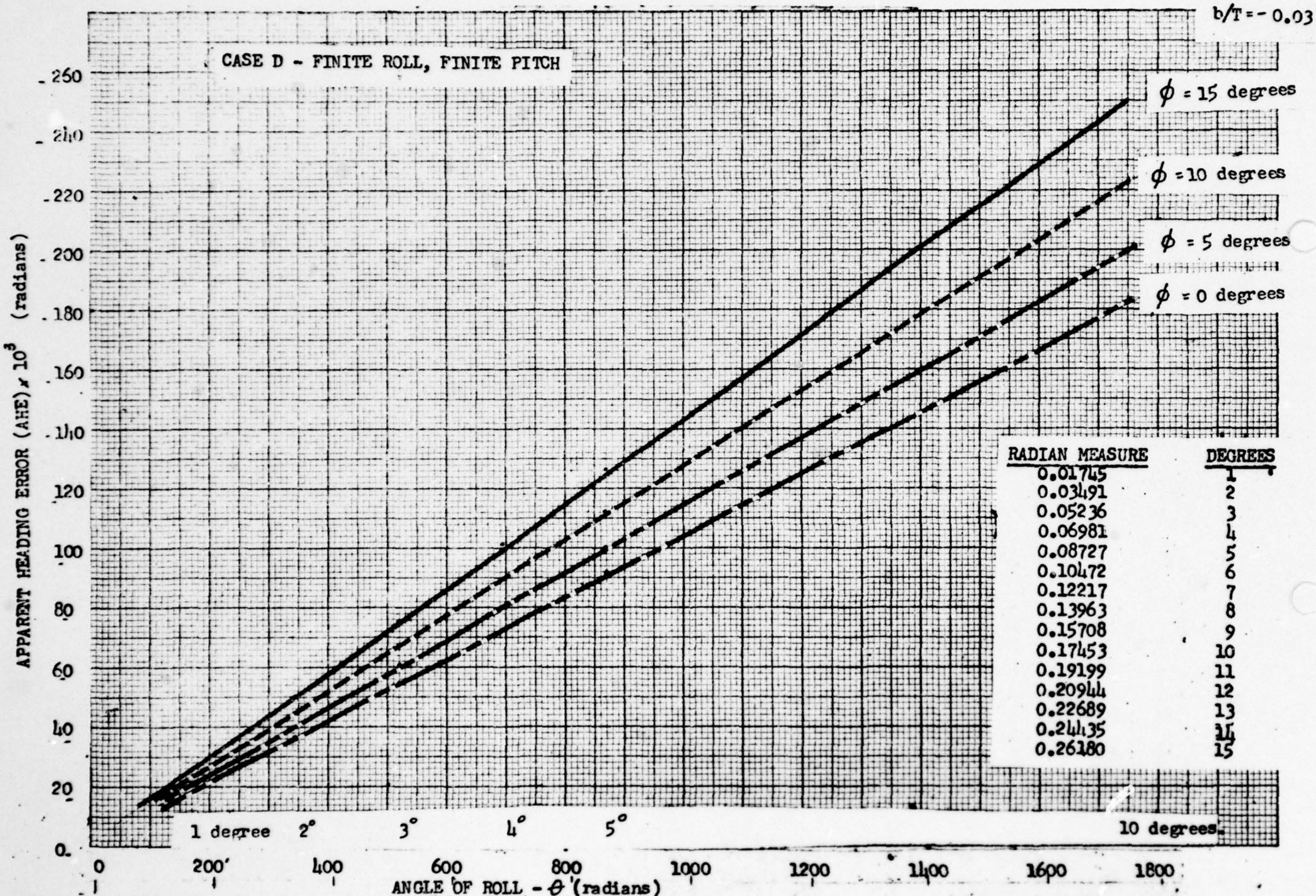
d/T = 1

a/T = + 0.03

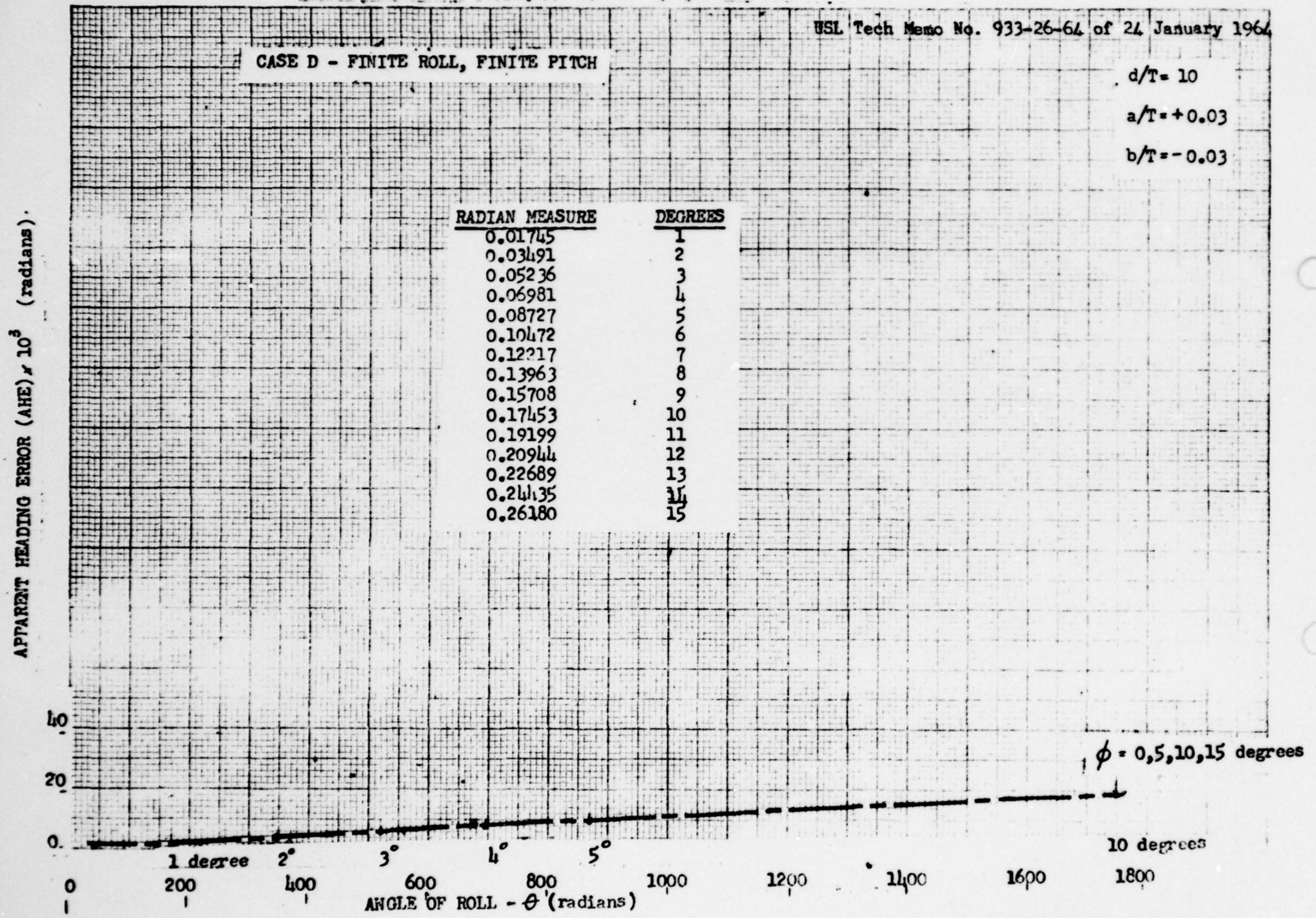
b/T = - 0.03



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

Figure 9 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

USL Tech Memo No. 933-26-64 of 24 January 1964



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

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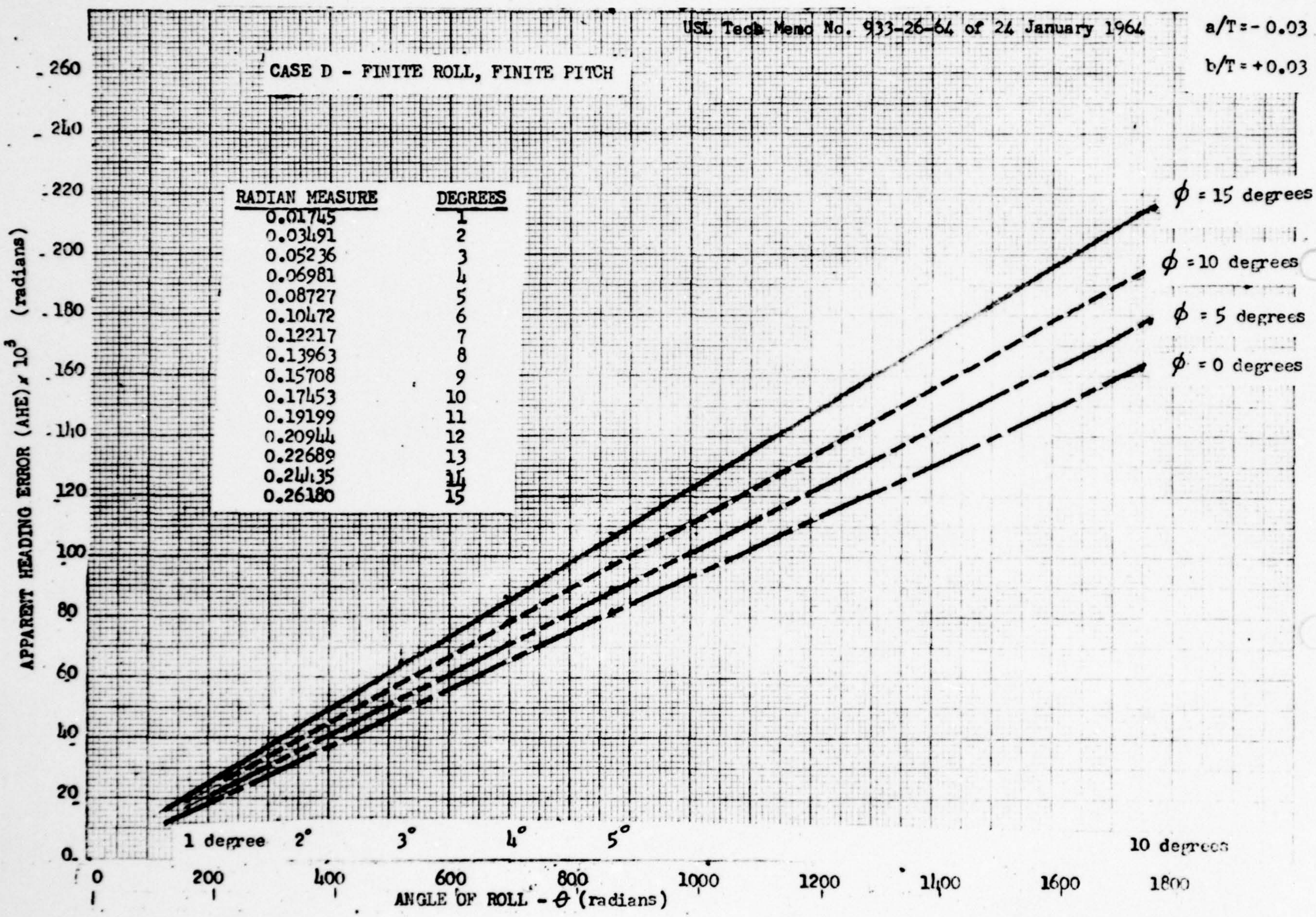
Figure 10 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

Figure 11 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

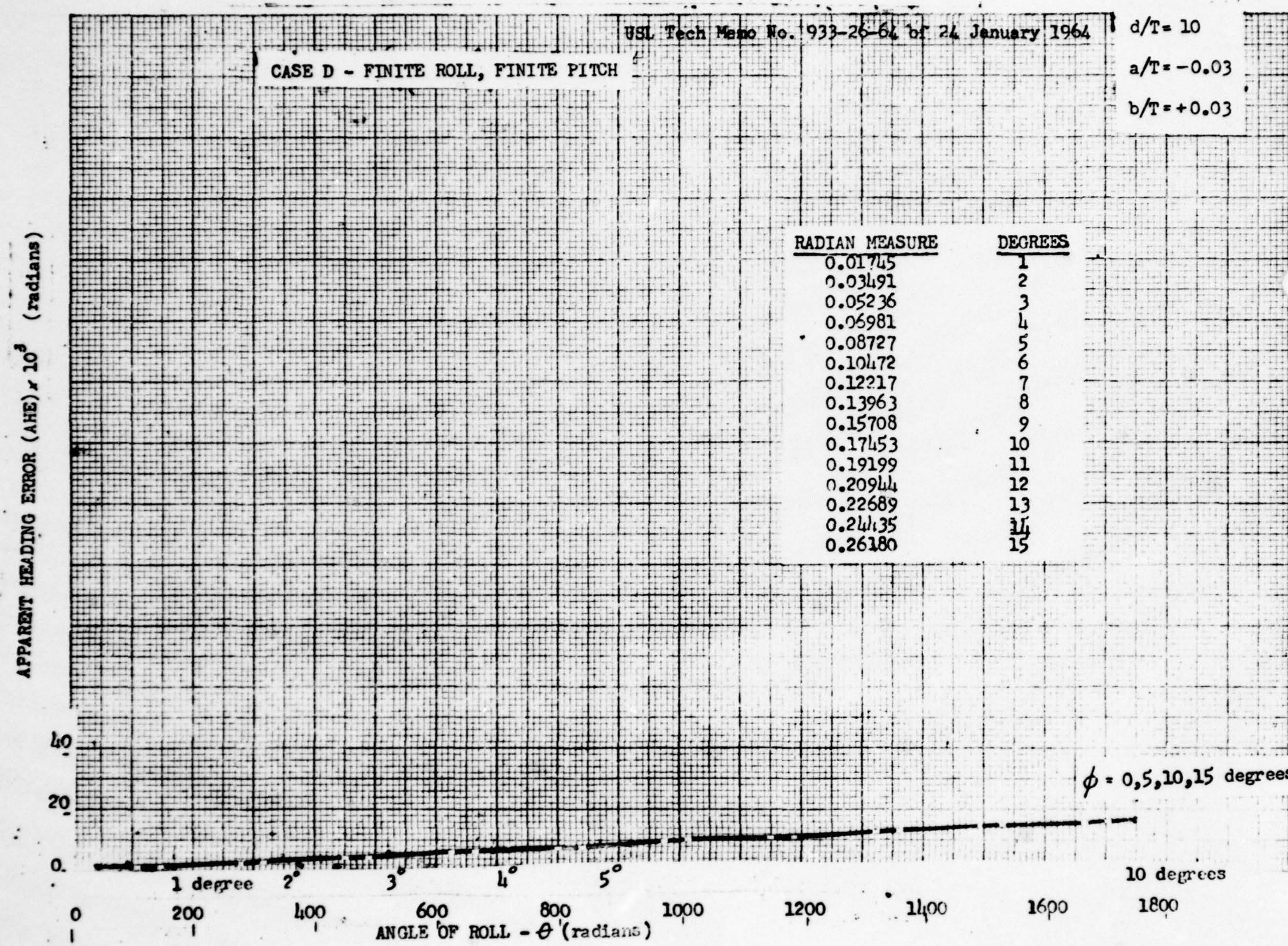


Figure 12 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

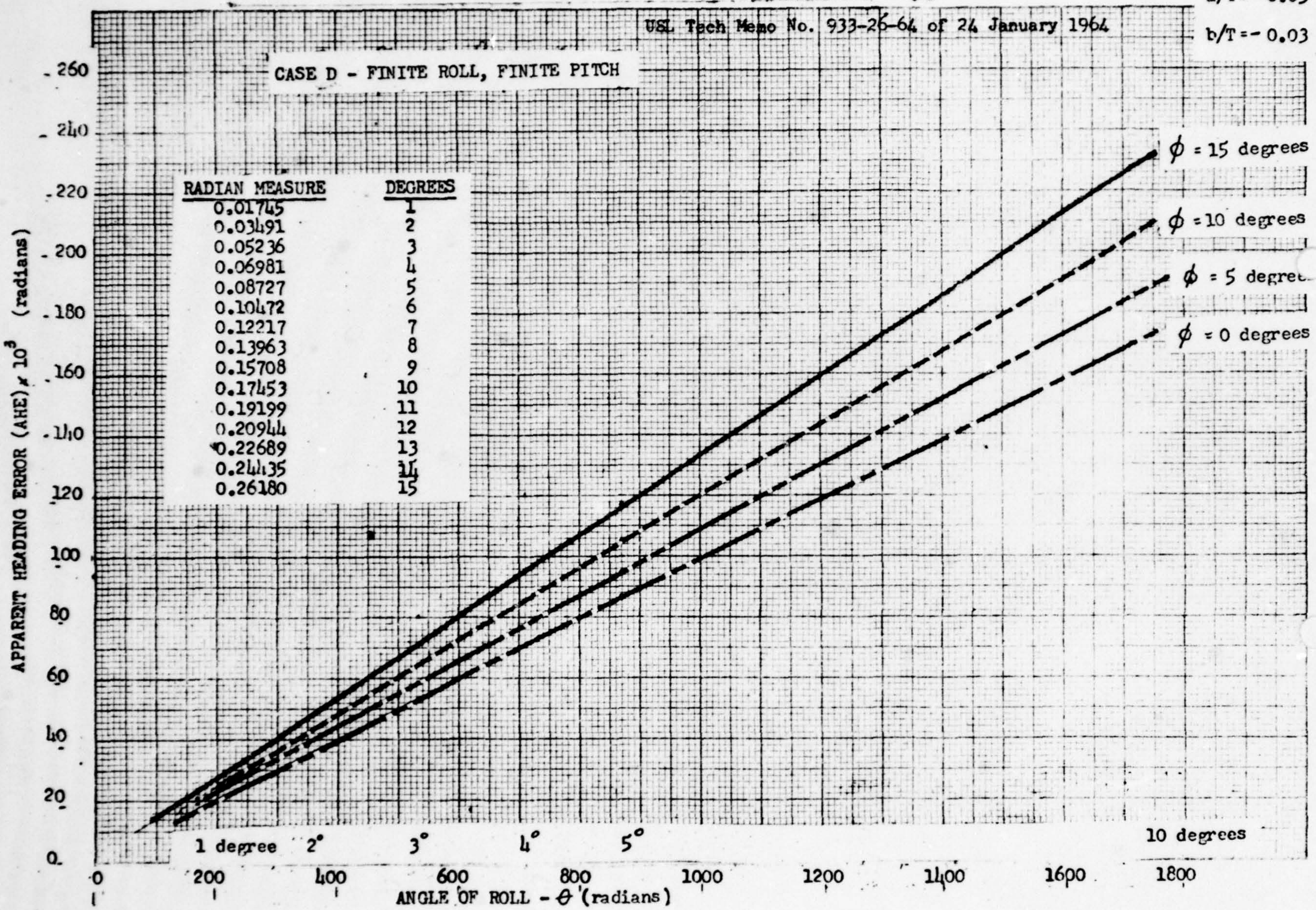


Figure 13 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

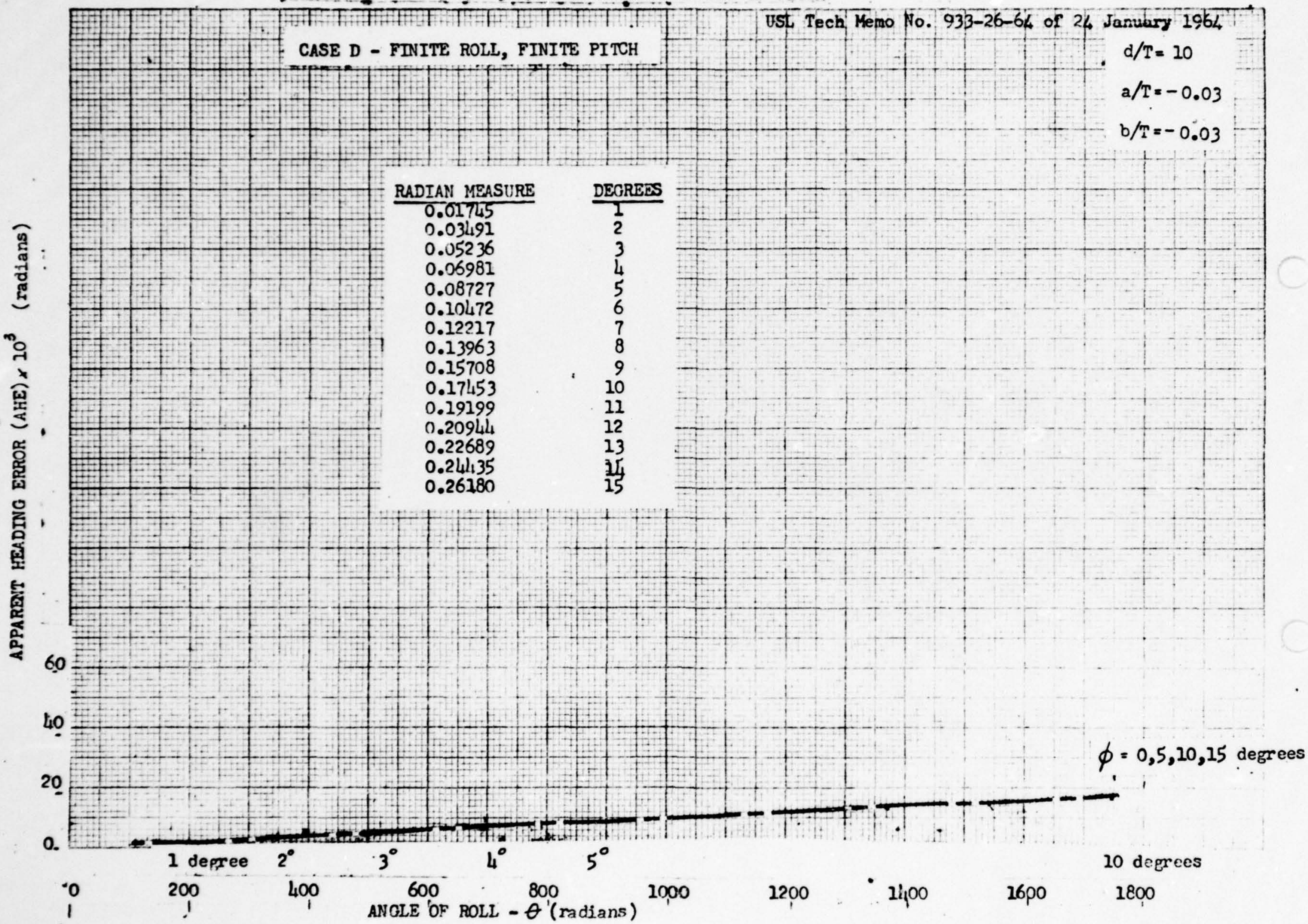
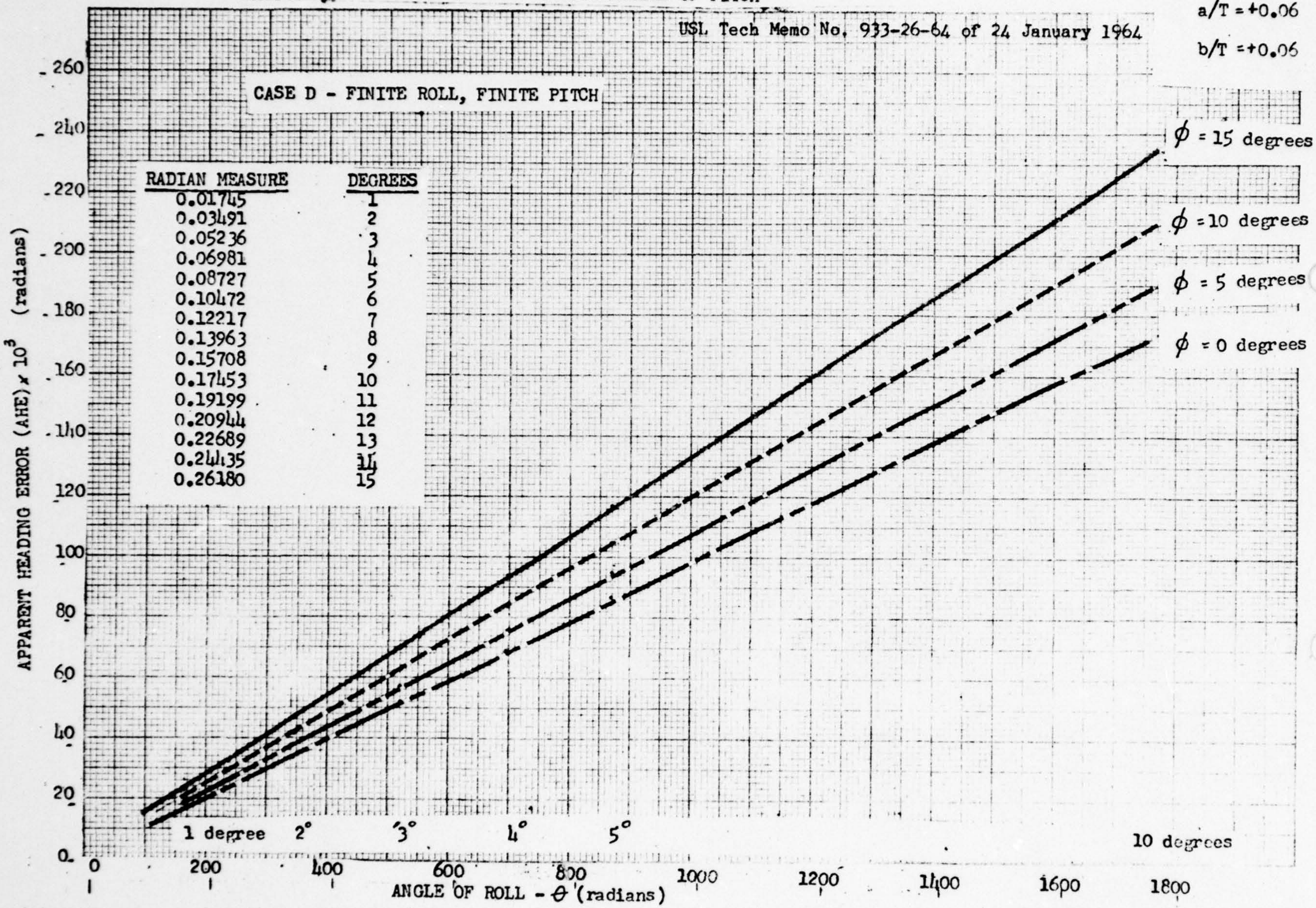


Figure 14 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

d/T = 1

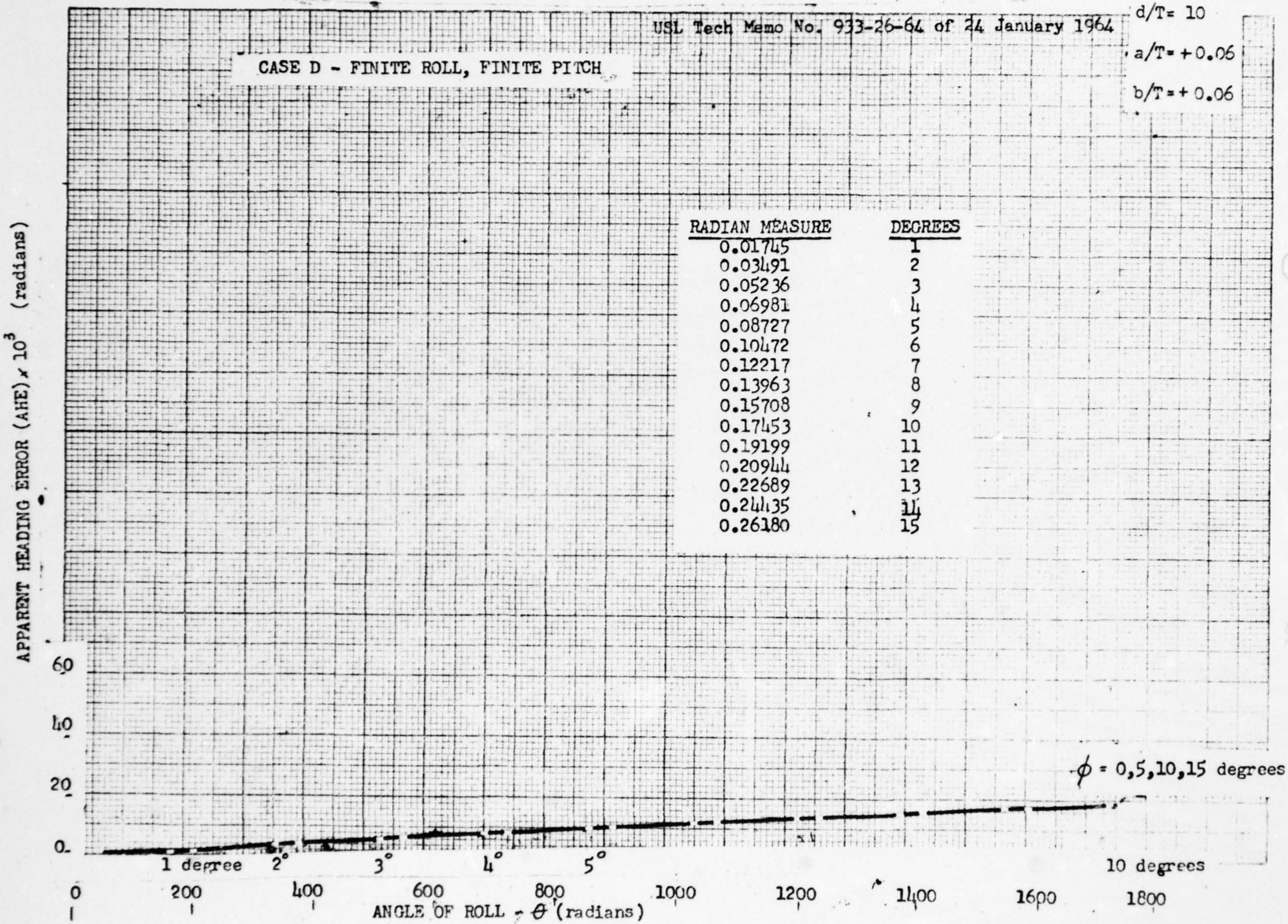
a/T = +0.06

b/T = +0.06



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

Figure 15 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15

Figure 16 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH

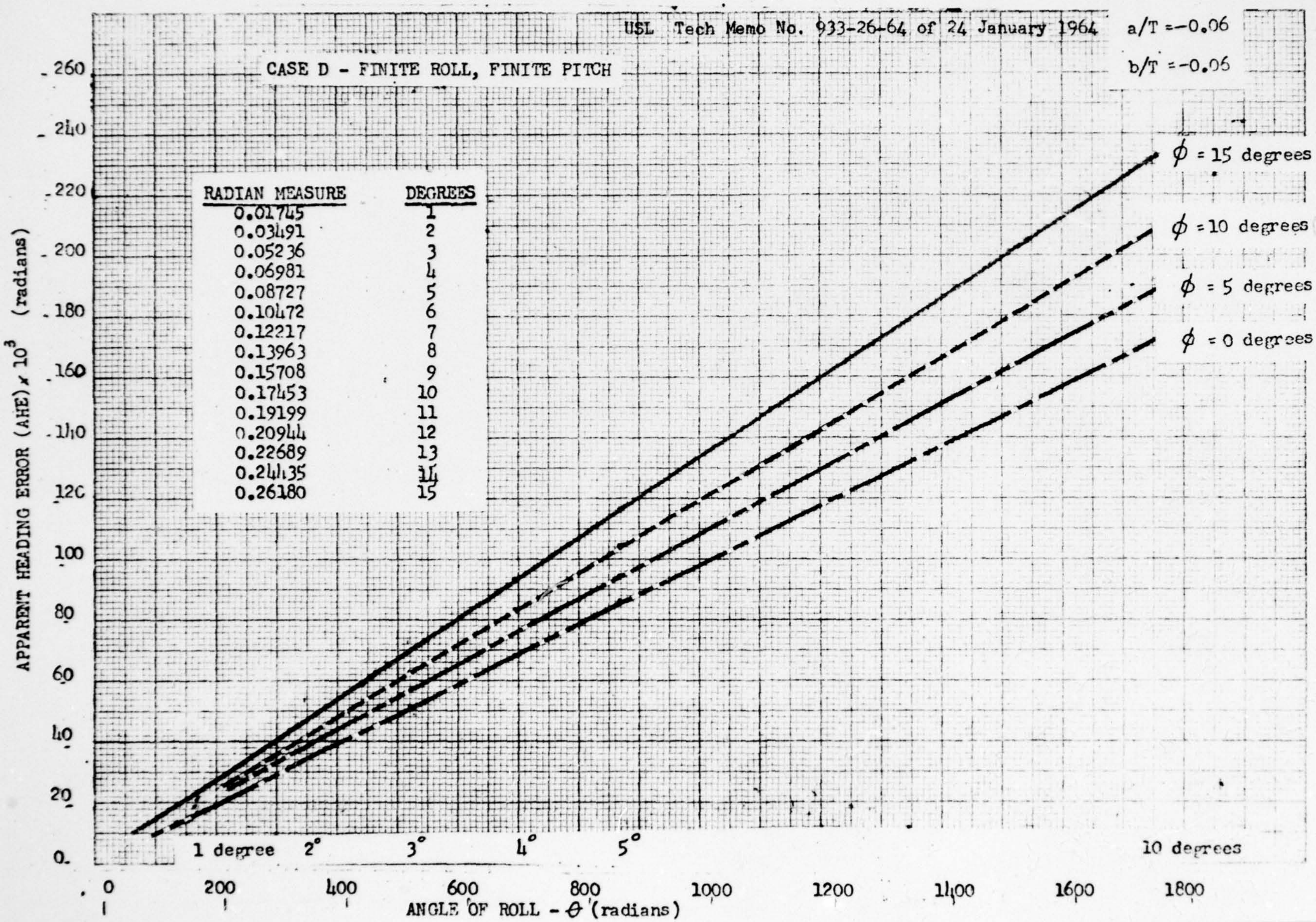
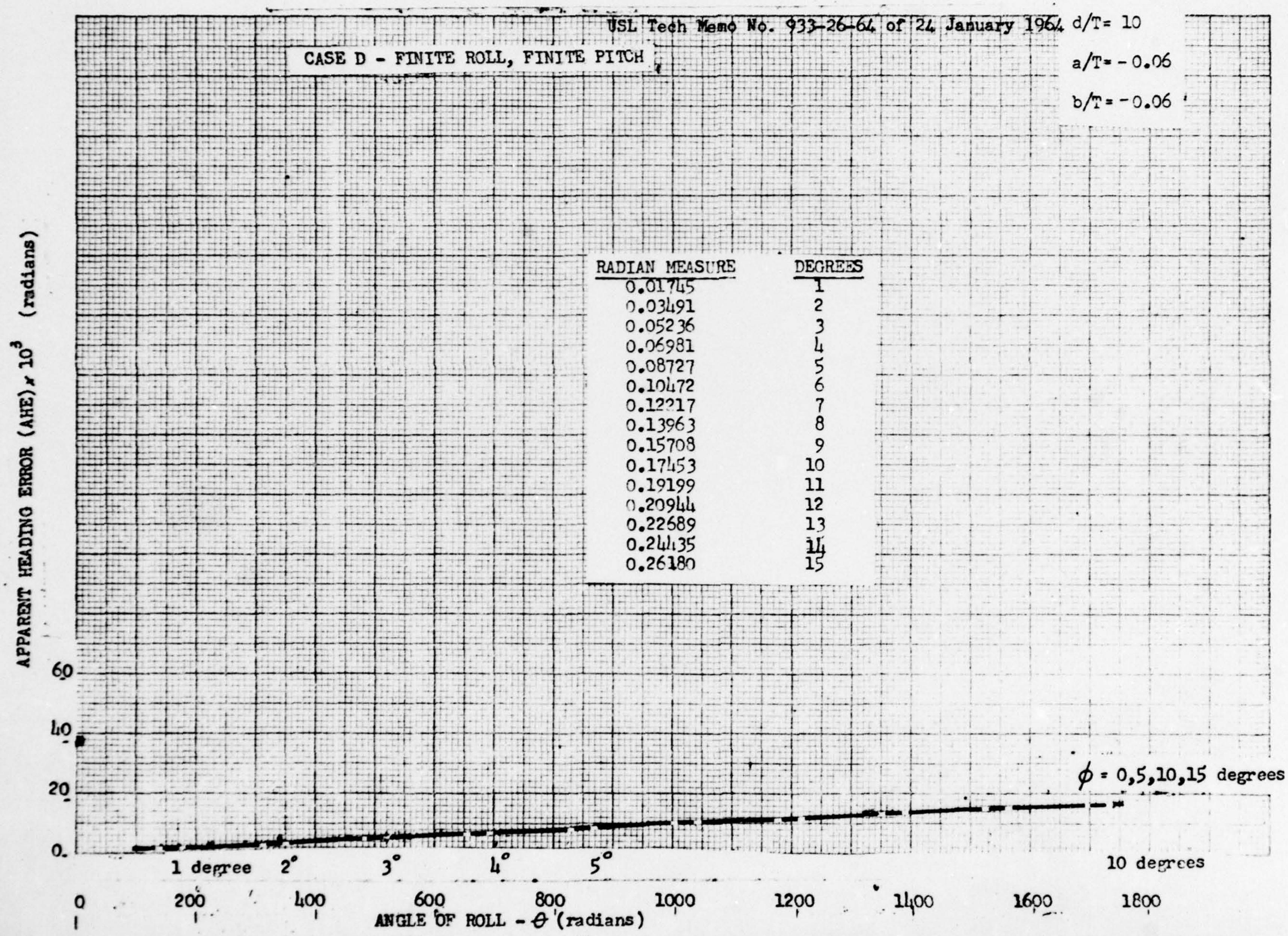


Figure 17 ANGLE OF ROLL versus APPARENT HEADING ERROR (AHE) at CONSTANT d/T ; a/T ; b/T for VARYING ANGLES OF PITCH



RADIAN MEASURE	DEGREES
0.01745	1
0.03491	2
0.05236	3
0.06981	4
0.08727	5
0.10472	6
0.12217	7
0.13963	8
0.15708	9
0.17453	10
0.19199	11
0.20944	12
0.22689	13
0.24435	14
0.26180	15